SECTION 13

OBSERVABLES

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13.1 INTRODUCTION

This section gives the formulation for the observed values and the computed values of spacecraft and quasar observables obtained by the Deep Space Network (DSN). The types of observables are doppler observables (which are described in Section 13.3), total-count phase observables (Section 13.4), range observables (Section 13.5), GPS/TOPEX observables (Section 13.6), spacecraft interferometry observables (Section 13.7), quasar interferometry observables (Section 13.8), and angular observables (Section 13.9). Each of these sections contains two parts. The first part contains the formulation for calculating the observed value of the observable from measured quantities obtained from the DSN. The second part contains the formulation for calculating the computed value of the observable. The definition of each observable applies for the observed and computed values of the observable.

Given spacecraft and quasar measurements obtained by the DSN, the observed values of the observables are calculated in the Orbit Data Editor (ODE). This is a generic name for whichever program is currently being used to perform the ODE function. The ODE writes the OD file, which is read by program Regres. The data record for each data point contains ID information which is necessary to unambiguously identify the data point (*e.g.*, the time tag, the data type, the transmitter, the spacecraft, the receiver, the doppler count time, band indicators, and constant frequencies), the observed value of the observable, and ancillary data needed by Regres to calculate the computed value of the observable (*e.g.*, down-leg and up-leg delays). Ramp records contain ramp tables (defined in Section 13.2) that specify the ramped transmitted frequency as a function of time at the various transmitting stations, and the ramped transmitter frequency (not transmitted) as a function of time, which is used as a doppler reference frequency (defined in Section 13.2) at various receiving stations. Phase records contain phase tables (defined in Section 13.2) that contain phase-time points which specify the phase of the transmitted signal as a function of time at the various transmitting stations.

Program Regres reads the OD file written by the ODE, calculates the computed values of the observables, and writes these quantities and related quantities which it calculates onto the Regres file. Each data record on the Regres file contains the OD file record for the data point, the computed value of the observable, the observed minus computed residual (RESID), the correction to the computed observable due to media corrections calculated in the Regres editor (CRESID) (see Section 10), the calculated weight for the data point (which is the inverse of the square of the calculated standard deviation for the data point), calculated auxiliary angles (see Section 9), the calculated partial derivatives of the computed observable with respect to the solve-for and consider parameters, and other quantities.

Calculation of the observed values of the observables (in the ODE) and/or the computed values of the observables (in program Regres) requires the time history of the transmitted frequency at the transmitter and related frequencies. The forms and sources of these frequencies are described in Section 13.2. Section 13.2.1 describes the transmitted frequency at a tracking station on Earth. This frequency can be constant or ramped. The ramped frequencies can be obtained from ramp tables or phase tables. Spacecraft turnaround ratios (the ratio of the transmitted to the received frequency at the spacecraft) are described in Section 13.2.2. If the transmitter is the spacecraft, the constant frequency transmitted at the spacecraft is described in Section 13.2.3. The doppler reference frequency, which is the transmitter frequency at the receiving station multiplied by a spacecraft turnaround ratio built into the electronics at the receiving station, is described in Section 13.2.4. Quasar frequencies are described in Section 13.2.5. Ramp tables and phase tables and the interpolation of them are described in Sections 13.2.6 and 13.2.7. The algorithm for the transmitted frequency on each leg of each light path is given in Section 13.2.8. This algorithm is used in calculating charged-particle corrections and other frequency-dependent terms.

As stated above, the formulations for calculating the observed and computed values of the various data types are given in Sections 13.3 to 13.9.

13.2 TRANSMITTER FREQUENCIES AND SPACECRAFT TURNAROUND RATIOS

13.2.1 TRANSMITTER FREQUENCY AT TRACKING STATION ON EARTH

The transmitter frequency $f_{\mathrm{T}}(t)$ at a tracking station on Earth can be constant or ramped. If it is a constant frequency, it is obtained from the record of the OD file for the data point. If the transmitter frequency is ramped, it is specified as a series of contiguous ramps. Each ramp has a start time, an end time, the frequency *f* at the start time, and the constant derivative $\dot{f}$ of *f* (the ramp rate) which applies between the start time and the end time for the ramp. Section 13.2.6 describes the content of ramp tables and gives the equations for interpolating them for the transmitter frequency *f* and its time derivative $\dot{f}$ at the interpolation time *t*. The start and end times for each ramp are in station time ST at the transmitting station on Earth. The interpolation time is the transmission time in station time ST at the transmitting electronics at the tracking station on Earth. In the near future, ramp tables will be supplemented with phase tables, which will eventually replace ramp tables. Phase tables contain a sequence of phase-time points. Each point gives the phase of the transmitted signal at the corresponding value of station time ST at the tracking station on Earth. Section 13.2.7 describes the content of phase tables and gives the equations for interpolating them for the phase ϕ, frequency *f*, and ramp rate $\dot{f}$ of the transmitted signal at the transmission time *t* in station time ST at the transmitting electronics at the tracking station on Earth.

An S-band or X-band transmitter frequency can be calculated from the corresponding reference oscillator frequency $f_{\mathrm{q}}(t)$. This frequency has an approximate value of 22 MHz and can be constant or ramped. In the past, OD file records contained constant values of $f_{\mathrm{q}}(t)$ and ramp tables gave the ramped values of $f_{\mathrm{q}}(t)$. Constant values of $f_{\mathrm{q}}(t)$ on the OD file are converted to $f_{\mathrm{T}}(t)$ in program Regres using the following equations:

For an S-band transmitted signal,

$$f_{\mathrm{T}}(t) = 96\, f_{\mathrm{q}}(t) \tag{13–1}$$

For an X-band uplink,

$$f_{\mathrm{T}}(t) = 32 f_{\mathrm{q}}(t) + 6.5 \times 10^{9} \text{ Hz} \tag{13–2}$$

For 34-m AZ-EL mount high efficiency X-band uplink antennas at DSS 15, 45, and 65 (prior to being converted to Block 5 receivers), the constant reference oscillator frequency reported was f_{q}' instead of f_{q}, and

$$f_{\mathrm{T}} = \frac{749}{5}\left(f_{\mathrm{q}}' + 26 \times 10^{6}\,\text{Hz}\right) \tag{13–3}$$

Equating Eqs. (13–2) and (13–3) gives f_{q} as the following exact function of f_{q}':

$$f_{\mathrm{q}} = 4.68125\, f_{\mathrm{q}}' - 81.4125 \times 10^{6} \text{ Hz} \tag{13–4}$$

The ODE uses Eq. (13–4) to convert the constant value of f_{q}' to f_{q}, which is placed on the OD file. Regres converts this value of f_{q} to $f_{\mathrm{T}}(t)$ using Eq. (13–2).

When the OD file contains ramp tables for $f_{\mathrm{q}}(t)$, Regres converts them to ramp tables for $f_{\mathrm{T}}(t)$ using the following procedure. The value of $f_{\mathrm{q}}(t)$ at the beginning of each ramp is converted to $f_{\mathrm{T}}(t)$ using Eq. (13–1) or (13–2). The ramp rates for each ramp are converted from f_q rates to f_T rates using the time derivatives of Eq. (13–1) or (13–2). For an S-band uplink,

$$\dot{f}_T = 96\,\dot{f}_q \tag{13–5}$$

For an X-band uplink,

$$\dot{f}_T = 32\,\dot{f}_q \tag{13–6}$$

Note that when $f_T(t)$ was ramped, the ramp tables gave ramped values of $f_q(t)$, not ramped values of $f_q^{\prime}(t)$.

The uplink band at the transmitting station on Earth is obtained from the data record of the OD file for the data point.

13.2.2 SPACECRAFT TURNAROUND RATIOS

The parameter M_2 is the spacecraft transponder turnaround ratio, which is the ratio of the transmitted down-leg frequency at the spacecraft to the received up-leg frequency at the spacecraft. Note that these two frequencies are phase coherent. The turnaround ratio M_2 is a function of the uplink band at the transmitting station on Earth and the downlink band for the data point. Both of these bands are obtained from the data record for the data point on the OD file. The following table (13-1) contains standard DSN spacecraft turnaround ratios for S, X, and Ka uplink and downlink bands.

Table 13–1

Spacecraft Turnaround Ratio M_2

Uplink Band	Downlink Band		
	S	X	Ka
S	$\frac{240}{221}$	$\frac{880}{221}$	$\frac{3344}{221}$
X	$\frac{240}{749}$	$\frac{880}{749}$	$\frac{3344}{749}$
Ka	$\frac{240}{3599}$	$\frac{880}{3599}$	$\frac{3344}{3599}$

The spacecraft turnaround ratios used in program Regres are input in the 7 x 7 GIN file array BNDRAT (*i,j*), where the integers *i* and *j* refer to the uplink and downlink bands:

1 = S
2 = X
3 = L
4 = C
5 = Ka
6 = Ku
7 = unused

The ODP user must make sure that the spacecraft turnaround ratios for the spacecraft whose tracking data he is processing are in the BNDRAT (*i,j*) array on the GIN file. Many spacecraft use non-standard turnaround ratios. For instance, the Cassini spacecraft uses an Italian transponder for a Ka-band uplink and downlink. The turnaround ratio which must be input in BNDRAT (5,5) is 14/15, not the standard ratio of 3344/3599 shown above in Table 13–1.

13.2.3 TRANSMITTER FREQUENCY AT SPACECRAFT

For round-trip data types, the transmitter is a tracking station on Earth. In the future, the transmitter may be an Earth satellite. For one-way data types, the transmitter is the spacecraft. When the spacecraft is the transmitter, the frequency of the transmitted signal (for all one-way data types except GPS/TOPEX observables) is:

$$f_{\mathrm{T}}(t) = C_2 \, f_{\mathrm{S/C}} \tag{13–7}$$

where $f_{\mathrm{S/C}}$ is the S-band value of the spacecraft transmitter frequency and C_2 converts it to the transmitted frequency for the downlink band for the data point (obtained from the data record for the data point on the OD file). The definition of $f_{\mathrm{S/C}}$ is:

$f_{\mathrm{S/C}}$ = the modelled S-band value of the spacecraft transmitter frequency in cycles per TAI second (9192631770 cycles of an imaginary cesium atomic clock carried by the spacecraft), nominally 2300 MHz.

The S-band value of the spacecraft transmitter frequency is calculated from:

$$f_{\mathrm{S/C}} = f_{\mathrm{T}_0} + \Delta f_{\mathrm{T}_0} + f_{\mathrm{T}_1}(t - t_0) + f_{\mathrm{T}_2}(t - t_0)^2 \tag{13–8}$$

where

f_{T_0} = nominal value of $f_{\mathrm{S/C}}$, obtained from the data record for the data point on the OD file.

$\Delta f_{\mathrm{T}_0}, f_{\mathrm{T}_1}, f_{\mathrm{T}_2}$ = solve-for quadratic coefficients used to represent the departure of $f_{\mathrm{S/C}}$ from f_{T_0}. The quadratic coefficients are specified by time block with start time t_0. The current time t and t_0 are measured in seconds of coordinate time ET past J2000.

The following table (13-2) contains standard DSN values of the down-leg frequency multiplier C_2 for S, X, and Ka downlink bands for the data point.

Table 13–2

Downlink Frequency Multiplier C_2

Downlink Band	Frequency Multiplier
S	1
X	$\frac{880}{240}$
Ka	$\frac{3344}{240}$

The downlink frequency multipliers C_2 used in program Regres are input in the 7-dimensional vector SCBAND(j) on the GIN file, where j is the downlink band specified after Table 13–1. The ODP user must make sure that the downlink frequency multipliers for the spacecraft that he is tracking are in the SCBAND (j) array on the GIN file.

13.2.4 DOPPLER REFERENCE FREQUENCY

The doppler reference frequency $f_{\mathrm{REF}}(t_3)$ is generated at the receiving station on Earth for one-way doppler (F_1), two-way doppler (F_2), and three-way doppler (F_3). It can be a constant frequency or a ramped frequency. It is used in the ODE to produce the observed values of F_1, F_2, and F_3 observables. The doppler reference frequency is also used in program Regres to calculate the computed values of F_2 and F_3 observables. It is not used in Regres in calculating the computed values of F_1 observables.

Prior to the introduction of Block 5 receivers, the doppler reference frequency was a real frequency. Block 5 receivers do not have a doppler reference frequency. However, at the current time, a fictitious constant doppler reference frequency is used at Block 5 receivers in calculating the observed and computed values of F_1, F_2, and F_3 observables. This fictitious frequency cancels completely in forming the observed minus computed residual. The purpose of the fictitious doppler reference frequency is to make Block 5 receiver doppler data look like the previous doppler observables. As soon as the Network Simplification Program (NSP) is implemented, the fictitious doppler reference frequency for Block 5 receivers will be set to zero. This will change and simplify the equations for calculating the observed and computed values of F_1, F_2, and F_3 observables in the ODE and in program Regres.

The doppler reference frequency directly affects the observed and computed values of doppler observables (Section 13.3). It indirectly affects total-count phase observables (Section 13.4), which are doppler observables multiplied by the doppler count interval T_c. It also indirectly affects narrowband spacecraft interferometry (*INS*) observables (Section 13.7.1), which are differenced doppler observables.

The doppler reference frequency at the reception time t_3 in station time ST at the receiving electronics at the receiving station on Earth is given by:

$$f_{\mathrm{REF}}(t_3) = M_{2_\mathrm{R}}\; f_\mathrm{T}(t_3) \tag{13–9}$$

The transmitter frequency $f_\mathrm{T}(t_3)$ at the reception time t_3 at the receiving electronics at the receiving station on Earth is a function of the uplink band at the receiving station on Earth. It can be constant or ramped and is calculated or obtained as described in Section 13.2.1. The quantity M_{2_R} is a spacecraft turnaround ratio built into the electronics at the receiving station on Earth. It is obtained from the GIN file as a function of the uplink band at the receiving station on Earth and the downlink band for the data point (see Section 13.2.2). The data record for the data point on the OD file contains the uplink band at the transmitting station on Earth, the uplink band at the receiving station on Earth, and the downlink band for the data point. It also contains a level indicator flag, which indicates whether $f_{\mathrm{REF}}(t_3)$ is calculated from constant or ramped values of the reference oscillator frequency $f_{\mathrm{q}}(t_3)$ (level 0), constant or ramped values of the transmitter frequency $f_{\mathrm{T}}(t_3)$ (level 1), or a constant value of $f_{\mathrm{REF}}(t_3)$ (level 2). The data record for the data point on the OD file contains the constant value of $f_{\mathrm{REF}}(t_3)$, specified at level 0, 1, or 2. If $f_{\mathrm{REF}}(t_3)$ is ramped, the ramp records of the OD file contain ramp tables for $f_{\mathrm{q}}(t_3)$ or $f_{\mathrm{T}}(t_3)$ at the receiving station on Earth. The data record for the data point on the OD file contains the simulation synthesizer flag, which specifies whether $f_{\mathrm{REF}}(t_3)$ is constant or ramped.

13.2.5 QUASAR FREQUENCIES

Narrowband quasar interferometry (*INQ*) observables are derived from the signal from a quasar received on a single channel at two receivers. Each of the two receivers can be a tracking station on Earth or an Earth satellite. The effective frequency of the quasar for a specific channel and pass is denoted as $\overline{\omega}$. For *INQ* observables, the quasar frequency $\overline{\omega}$ is placed on the data record of the OD file for the data point.

Wideband quasar interferometry (*IWQ*) observables are derived from the signal from a quasar received on two channels at two receivers. The effective frequencies of the quasar for channels B and A are denoted as $\overline{\omega}_{\mathrm{B}}$ and $\overline{\omega}_{\mathrm{A}}$, respectively. For *IWQ* observables, the average quasar frequency $(\overline{\omega}_{\mathrm{B}} + \overline{\omega}_{\mathrm{A}})/2$ is placed on the data record of the OD file for the data point.

13.2.6 RAMP TABLES

The ramp table for a given tracking station gives the value of the ramped transmitter frequency $f_{\mathrm{T}}(t)$ and its time derivative $\dot{f}$ at the interpolation time t. The interpolation time t is the transmission time in station time ST at the transmitting electronics at the tracking station on Earth. Each ramp in the ramp table is specified by four numbers. They are the start time t_{o} and end time t_{f} of the ramp in station time ST at the tracking station (integer seconds), the value of the ramped transmitter frequency f_{o} at the start time t_{o} of the ramp, and the constant time derivative (the ramp rate) $\dot{f}$ of $f_{\mathrm{T}}(t)$ which applies from t_{o} to t_{f}. The value of $f_{\mathrm{T}}(t)$ at the interpolation time t is given by:

$$f_{\mathrm{T}}(t) = f_{\mathrm{o}} + \dot{f}\left(t - t_{\mathrm{o}}\right) \tag{13–10}$$

The ramp table for the transmitting station gives the ramped transmitted frequency $f_{\mathrm{T}}(t)$ as a function of time. For doppler observables, the ramp table for the receiving station gives the ramped transmitter frequency $f_{\mathrm{T}}(t)$ at the receiving station as a function of time. This ramped frequency or an alternate constant value of $f_{\mathrm{T}}(t)$ at the receiving station can be used to calculate the doppler reference frequency $f_{\mathrm{REF}}(t_3)$ from Eq. (13–9).

Ramp tables can be specified at the reference oscillator frequency $f_{\mathrm{q}}(t)$ level or at the transmitter frequency $f_{\mathrm{T}}(t)$ level. The former type of ramp table can be converted to the latter type of ramp table as described in Section 13.2.1.

In the future, tracking stations on Earth will be transmitting simultaneously at two different frequency bands (*e.g.*, X-band and Ka-band). When this occurs, ramp tables will have to be labelled with the transmitting station and the uplink band. Currently, the uplink band is not included in ramp tables.

13.2.7 PHASE TABLES

In the near future, ramp tables will be replaced with phase tables. Phase tables contain a sequence of phase-time points. Each point gives the (quadruple precision) phase of the transmitted signal at the corresponding value of station time ST (integer seconds) at a particular tracking station on Earth. The uplink band at the tracking station should be added to the phase table, since tracking stations will be transmitting simultaneously at two different frequency bands in the not too distant future.

Interpolation of the phase table for a particular tracking station on Earth and uplink band gives the phase ϕ and frequency f of the transmitted signal and the constant time derivative $\dot{f}$ (the ramp rate) of the transmitted frequency at the interpolation time t, which is the transmission time in station time ST at the transmitting electronics at the tracking station on Earth. Interpolation of the phase table requires three phase-time pairs on the same ramp: ϕ_1 at t_1, ϕ_2 at t_2, and ϕ_3 at t_3. The interpolation time t will be between t_1 and t_3, and it may be before or after t_2. The phase differences $\phi_2 - \phi_1$ and $\phi_3 - \phi_2$ can be expressed as a function of the frequency f_2 at t_2, the ramp rate $\dot{f}$ (which is constant from t_1 to t_3), and the time differences:

$$T_A = t_2 - t_1 \qquad \text{s} \qquad (13\text{–}11)$$

and

$$T_B = t_3 - t_2 \qquad \text{s} \qquad (13\text{–}12)$$

Solving these two equations for f_2 and $\dot{f}$ gives:

$$f_2 = \frac{1}{\left(T_A + T_B\right)}\left[\left(\phi_2 - \phi_1\right)\left(\frac{T_B}{T_A}\right) + \left(\phi_3 - \phi_2\right)\left(\frac{T_A}{T_B}\right)\right] \qquad \text{Hz} \qquad (13\text{–}13)$$

and

$$\dot{f} = \frac{2}{\left(T_{\mathrm{A}} + T_{\mathrm{B}}\right)} \left[\frac{\left(\phi_3 - \phi_2\right)}{T_{\mathrm{B}}} - \frac{\left(\phi_2 - \phi_1\right)}{T_{\mathrm{A}}} \right] \qquad \mathrm{Hz/s} \qquad (13\text{–}14)$$

The phase differences in these two equations should be calculated in quadruple precision and then rounded to double precision.

Define Δt to be the interpolation time t minus the time argument t_2 for the phase ϕ_2 obtained from the phase table:

$$\Delta t = t - t_2 \qquad \mathrm{s} \qquad (13\text{–}15)$$

Also, define $\Delta\phi\left(\Delta t\right)$ to be the phase $\phi(t)$ of the transmitted signal at the interpolation time t minus the phase obtained from the phase table at t_2:

$$\Delta\phi\left(\Delta t\right) = \phi(t) - \phi_2 \qquad \text{cycles} \qquad (13\text{–}16)$$

Given f_2 and $\dot{f}$, the phase difference that accumulates from the tabular time t_2 to the interpolation time t is given by:

$$\Delta\phi\left(\Delta t\right) = f_2\,\Delta t + \tfrac{1}{2}\,\dot{f}\,\left(\Delta t\right)^2 \qquad \text{cycles} \qquad (13\text{–}17)$$

Adding this phase difference to the tabular phase ϕ_2 obtained from the phase table at t_2 gives the phase of the transmitted signal at the interpolation time t. The transmitted frequency at the interpolation time t is given by:

$$f_{\mathrm{T}}(t) = f_2 + \dot{f}\,\Delta t \qquad \mathrm{Hz} \qquad (13\text{–}18)$$

13.2.8 ALGORITHM FOR TRANSMITTED FREQUENCY ON EACH LEG OF LIGHT PATH

This section gives the algorithm for calculating the transmitter frequency f on the up leg and down leg of the spacecraft light-time solution and on the down legs from a quasar to each of the two receivers. The frequency f is used to calculate charged-particle corrections (Section 10.2.2) and partial derivatives with respect to the N and D coefficients of the ionosphere model of Klobuchar (1975) (Section 10.3). It is also used to calculate solar corona corrections and partial derivatives with respect to the A, B, and C coefficients of the solar corona model (Section 10.4).

This algorithm does not apply for the down-leg spacecraft light-time solution used for GPS/TOPEX pseudo-range and carrier-phase observables. These observables are calculated as a weighted average of observables with L1-band and L2-band transmitter frequencies (see Eq. 7–1). The weighted average (Eqs. 7–2 to 7–4) was selected to eliminate the charged-particle effect from these data types. However, there are three remaining frequency-dependent corrections to these observables, namely, the geometrical phase correction for carrier-phase observables, constant phase-center offsets, and variable phase-center offsets for carrier-phase observables. These frequency-dependent corrections are computed as weighted averages of the L1-band and L2-band values of the corrections as described in Sections 7.3.1, 7.3.3, 8.3.6 (Step 9), 11.5.3, and 11.5.4.

The frequency f on the down legs to the two receivers for narrowband (*INQ*) and wideband (*IWQ*) quasar interferometry observables is described in Section 13.2.5. This frequency is obtained from the data record of the OD file for the data point.

The frequency f for the up leg of the spacecraft light-time solution is the transmitter frequency $f_{\mathrm{T}}(t_1)$ at the transmission time t_1 at the transmitter (a tracking station on Earth or an Earth satellite):

$$f = f_{\mathrm{T}}(t_1) \qquad \text{Hz} \qquad (13\text{–}19)$$

If the transmitter is a tracking station on Earth, the transmitter frequency $f_{\mathrm{T}}(t_1)$ at the transmission time t_1 is calculated as described in Sections 13.2.1, 13.2.6, and 13.2.7.

The frequency f for the down leg of the spacecraft light-time solution is the transmitter frequency $f_{\mathrm{T}}(t_1)$ multiplied by the spacecraft transponder turnaround ratio M_2, which is obtained as described in Section 13.2.2:

$$f = M_2 \; f_{\mathrm{T}}(t_1) \qquad \text{Hz} \qquad (13\text{–}20)$$

The down-leg frequency f given by Eq. (13–20) is required when performing the down leg of the spacecraft light-time solution. For each estimate of the transmission time t_2(ET) for the down leg of the light path, calculate the predicted up-leg light time $t_2 - t_1$ from Eqs. (8–79) and (8–80). Subtract $t_2 - t_1$ from t_2(ET) to give an estimate for the transmission time t_1(ET) for the up leg of the light path. Use t_1(ET) to calculate f for the down leg from Eq. (13–20). The up-leg frequency f given by Eq. (13–19) is required when performing the up leg of the spacecraft light-time solution. Eq. (13–19) is evaluated using the estimate of t_1(ET) which is available for each iteration of the up-leg light-time solution.

When the spacecraft is the transmitter, the frequency f for the down leg of the spacecraft light-time solution (for all one-way data types except GPS/TOPEX observables) is a simplified version of the transmitted frequency given by Eqs. (13–7) and (13–8):

$$f = C_2 \; f_{\mathrm{T}_0} \qquad \text{Hz} \qquad (13\text{–}21)$$

The right-hand side of this equation is calculated as described in Section 13.2.3.

13.3 DOPPLER OBSERVABLES

Section 13.3.1 gives the formulas used to calculate the observed values of one-way (F_1), two-way (F_2), and three-way (F_3) doppler observables in the ODE from measured quantities (frequencies and phases) obtained from the Deep Space Network (DSN). Observed and computed values of F_2 and F_3 doppler observables are calculated from the ramped doppler formulation or the unramped doppler formulation. F_1 doppler is always unramped. Section 13.3.1.1 applies for observables obtained from receivers older than Block 5 receivers (BVR). Section 13.3.1.2 applies for observables obtained from BVRs prior to implementation of the Network Simplification Program (NSP). Section 13.3.1.3 applies for observables obtained from BVRs after implementation of the NSP.

Section 13.3.2 gives the formulas used to calculate the computed values of F_1, F_2, and F_3 doppler observables in program Regres. Subsections 13.3.2.1, 13.3.2.2, and 13.3.2.3 apply for unramped F_2 and F_3 observables, ramped F_2 and F_3 observables, and F_1 observables, respectively. Each section gives the formulation for the computed value of the observable, the correction to the computed value of the observable due to media corrections (calculated in the Regres editor), and partial derivatives of the computed observable with respect to solve-for and consider parameters. Variations in these formulations which apply for receivers older than BVRs, BVRs prior to the NSP, and BVRs after implementation of the NSP are given.

13.3.1 OBSERVED VALUES OF DOPPLER OBSERVABLES

13.3.1.1 Observables Obtained From Receivers Older Than Block 5 Receivers

The signal input to the doppler counter at the receiving station on Earth has the frequency $f(t_3)$ in cycles per second of station time ST. The time argument t_3 is the reception time in station time ST at the receiving electronics.

$$f(t_3) = f_{\mathrm{REF}}(t_3) - f_{\mathrm{R}}(t_3) + C_4 \qquad \mathrm{Hz} \qquad (13\text{–}22)$$

where

$f_{\mathrm{REF}}(t_3)$ = doppler reference frequency at reception time t_3 at receiving station on Earth. See Section 13.2.4.

$f_{\mathrm{R}}(t_3)$ = frequency of received signal at reception time t_3 at receiving station on Earth.

C_4 = constant bias frequency (normally $\pm 1\,\mathrm{MHz}$) generated at receiving station on Earth.

The doppler counter measures cycles of $f(t_3)$ that accumulate from an epoch t_{3_0} near the start of the pass to the current time t_3:

$$N(t_3) = \int_{t_{3_0}}^{t_3} f(t_3)\, dt_3 \qquad \text{cycles} \qquad (13\text{–}23)$$

The accumulated doppler cycle count $N(t_3)$ is available every 0.1 second of station time ST at the receiving electronics at the receiving station on Earth.

Doppler observables are derived from the change in the doppler cycle count $N(t_3)$, which accumulates during the count interval or count time T_c at the receiving station on Earth. Successive doppler observables at a given tracking station on Earth have contiguous count intervals. Count intervals can be as short as 0.1 s (very rare) or as long as a pass of data (about half a day or 43,200 s) (also very rare). Typical count times have durations of tens of seconds to a few thousand seconds. Shorter count times are used at encounters with celestial bodies and longer count times are used in interplanetary cruise. Count intervals of 1 s or longer are integer seconds and begin and end on seconds pulses. Count intervals less than 1 s are integer tenths of a second and begin and end on tenths of a second pulses. The time tag TT of a doppler observable is the midpoint of the count interval T_c. The time tag ends in integer seconds, tenths of a second, or hundredths of a second. Given the time tag TT and count interval T_c for a doppler observable, the epochs at the start and end of the count interval are given by:

$$t_{3_e}(\mathrm{ST})_R = TT + \tfrac{1}{2}T_c \qquad \text{s} \qquad (13\text{–}24)$$

$$t_{3_s}(\mathrm{ST})_R = TT - \tfrac{1}{2}T_c \qquad \text{s} \qquad (13\text{–}25)$$

where these epochs, the time tag TT, and the count time T_c are measured in seconds of station time ST at the receiving electronics (subscript R) at the receiving station on Earth.

Observed values of all doppler observables are calculated in the ODE from:

$$F = \frac{\Delta N}{T_c} - f_{\mathrm{bias}} \qquad \text{Hz} \qquad (13\text{–}26)$$

where F can be one-way doppler (F_1), unramped two-way (F_2) or three-way (F_3) doppler, or ramped F_2 or F_3. The quantity ΔN is the change in the doppler cycle count $N(t_3)$ given by Eq. (13–23), which accumulates during the count interval T_c:

$$\Delta N = N\left(t_{3_e}\right) - N\left(t_{3_s}\right) \qquad \text{cycles} \qquad (13\text{–}27)$$

where $N\left(t_{3_e}\right)$ and $N\left(t_{3_s}\right)$ are values of $N(t_3)$ given by Eq. (13–23) at the epochs given by Eqs. (13–24) and (13–25), respectively. Since values of $N(t_3)$ are given every 0.1 s, no interpolation of this data is required. The equation for calculating the bias frequency f_{bias} depends upon the data type.

For one-way doppler (F_1), the bias frequency f_{bias} is calculated from:

$$f_{\mathrm{bias}} = f_{\mathrm{REF}}\left(t_3\right) - C_2 f_{T_0} + C_4 \qquad \text{Hz} \qquad (13\text{–}28)$$

where the downlink frequency multiplier C_2 and the nominal value f_{T_0} of the spacecraft transmitter frequency at S-band are described in Section 13.2.3. Eq. (13–28) removes the effect of the departure of the constant $f_{\mathrm{REF}}\left(t_3\right)$ from $C_2 f_{T_0}$ and the effect of C_4 from the one-way doppler observable calculated from Eqs. (13–22) to (13–28) in the ODE. From these equations, the definition of the one-way doppler observable calculated in the ODE is given by:

$$F_1 = \frac{1}{T_c} \int_{t_{3_s}(\mathrm{ST})_R}^{t_{3_e}(\mathrm{ST})_R} \left[C_2 f_{T_0} - f_R(t_3) \right] dt_3 \qquad \mathrm{Hz} \qquad (13\text{–}29)$$

Since $C_2 f_{T_0}$ is the transmitter frequency at the spacecraft, the one-way doppler observable calculated in the ODE is the negative of the average doppler frequency shift which occurs over the count interval T_c.

For unramped two-way (F_2) or three-way (F_3) doppler, the bias frequency f_{bias} is calculated from:

$$f_{bias} = f_{REF}(t_3) - M_2 f_T(t_1) + C_4 \qquad \mathrm{Hz} \qquad (13\text{–}30)$$

where the spacecraft transponder turnaround ratio M_2 and the constant transmitter frequency $f_T(t_1)$ at the transmitting station on Earth are described in Sections 13.2.2 and 13.2.1, respectively. Eq. (13–30) removes the effect of the departure of the constant $f_{REF}(t_3)$ from the effective transmitter frequency $M_2 f_T(t_1)$ and the effect of C_4 from the unramped two-way (F_2) or three-way (F_3) doppler observable calculated from Eqs. (13–22) to (13–27) and Eq. (13–30) in the ODE. From these equations, the definition of the unramped two-way (F_2) or three-way (F_3) doppler observable calculated in the ODE is given by:

$$\text{unramped } F_{2,3} = \frac{1}{T_c} \int_{t_{3_s}(\mathrm{ST})_R}^{t_{3_e}(\mathrm{ST})_R} \left[M_2 f_T(t_1) - f_R(t_3) \right] dt_3 \qquad \mathrm{Hz} \qquad (13\text{–}31)$$

Since $M_2 f_T(t_1)$ is the effective transmitter frequency at the tracking station on Earth, the unramped two-way (F_2) or three-way (F_3) doppler observable calculated in the ODE is the negative of the average doppler frequency shift which occurs over the count interval T_c.

For ramped two-way (F_2) or three-way (F_3) doppler, the transmitter frequency $f_T(t_1)$ at the transmitting station on Earth is ramped, and the doppler reference frequency $f_{REF}(t_3)$ at the receiving station may be ramped or constant. The difference of these two frequencies is not constant and its effect on computed ramped two-way (F_2) or three-way (F_3) doppler observables calculated in the ODE cannot be removed. Hence, for these observables, the bias frequency f_{bias} is given by:

$$f_{bias} = C_4 \qquad \text{Hz} \qquad (13\text{–}32)$$

From Eqs. (13–22) to (13–27) and Eq. (13–32), the definition of the ramped two-way (F_2) or three-way (F_3) doppler observable calculated in the ODE is given by:

$$\text{ramped } F_{2,3} = \frac{1}{T_c} \int_{t_{3_s}(\mathrm{ST})_R}^{t_{3_e}(\mathrm{ST})_R} \left[f_{REF}(t_3) - f_R(t_3) \right] dt_3 \qquad \text{Hz} \qquad (13\text{–}33)$$

Because the doppler reference frequency $f_{REF}(t_3)$ is not the same as the effective transmitter frequency $M_2 f_T(t_1)$, the ramped two-way (F_2) or three-way (F_3) doppler observable calculated in the ODE is not equal to the negative of the average doppler frequency shift which occurs over the count interval T_c.

13.3.1.2 Observables Obtained From Block 5 Receivers Before Implementation of Network Simplification Program (NSP)

Block 5 receivers do not produce the doppler cycle count $N(t_3)$ given by Eqs. (13–22) and (13–23). Instead, they count cycles of the received frequency $f_R(t_3)$, which accumulate from an epoch t_{3_0} near the start of the pass to the current time t_3:

$$\phi(t_3) = \int_{t_{3_0}}^{t_3} f_R(t_3)\, dt_3 \qquad \text{cycles} \qquad (13\text{–}34)$$

The accumulated phase $\phi(t_3)$ of the received signal is measured every 0.1 s of station time ST at the receiving electronics at the receiving station on Earth. Also, Block 5 receivers do not have a doppler reference signal. As an interim procedure which will be used until the NSP is completed, the Metric Data Assembly (MDA) obtains the accumulated phase $\phi(t_3)$ of the received signal at the reception time t_3 at the receiving electronics and creates the following doppler cycle count $N(t_3)$ using Eqs. (13–22), (13–23), and (13–34):

$$N\left(t_3\right) = f_{\text{REF}}\left(t_3\right)\left(t_3 - t_{3_0}\right) - \phi\left(t_3\right) + C_4\left(t_3 - t_{3_0}\right) \qquad \text{cycles} \qquad (13\text{–}35)$$

The doppler reference frequency $f_{\text{REF}}\left(t_3\right)$ is a constant fictitious frequency created in the MDA. It is specified at level 0, 1, or 2 as described in Section 13.2.4.

Given $N(t_3)$ calculated from Eq. (13–35) in the MDA for every 0.1 s of station time ST at the receiving electronics at the receiving station on Earth, observed values of doppler observables are calculated in the ODE from Eqs. (13–24) to (13–27). The bias frequency f_{bias} is calculated from Eq. (13–28) for one-way doppler (F_1) and from Eq. (13–32) for ramped two-way (F_2) or three-way (F_3) doppler observables. Note that Block 5 receivers do not produce unramped two-way (F_2) or three-way (F_3) doppler observables.

13.3.1.3 Observables Obtained From Block 5 Receivers After Implementation of Network Simplification Program (NSP)

After the Network Simplification Program (NSP) is implemented, the doppler reference frequency $f_{\text{REF}}\left(t_3\right)$ and the bias frequency C_4 will be set to zero. With these changes, the doppler cycle count $N(t_3)$ given by Eqs. (13–22), (13–23), and (13–34) or Eqs. (13–34) and (13–35) reduces to:

$$N\left(t_3\right) = -\ \phi\left(t_3\right) \qquad \text{cycles} \qquad (13\text{–}36)$$

For ramped two-way (F_2) or three-way (F_3) doppler observables, the bias frequency f_{bias} given by Eq. (13–32) reduces to:

$$f_{\text{bias}} = 0 \qquad \text{Hz} \qquad (13\text{–}37)$$

Substituting Eqs. (13–36) and (13–37) into Eqs. (13–24) to (13–27) gives the following equation for calculating the observed values (F) of ramped two-way (F_2) or three-way (F_3) doppler observables in the ODE:

$$F = -\frac{\left[\phi\left(t_{3_e}\right) - \phi\left(t_{3_s}\right)\right]}{T_c} \qquad \text{Hz} \qquad (13\text{–}38)$$

Setting $f_{\text{REF}}(t_3)$ and C_4 equal to zero in Eq. (13–28) for f_{bias} for one-way doppler (F_1) observables gives:

$$f_{\text{bias}} = -\,C_2\, f_{T_0} \qquad \text{Hz} \qquad (13\text{–}39)$$

Substituting Eqs. (13–36) and (13–39) into Eqs. (13–24) to (13–27) gives the following equation for calculating the observed values of one-way (F_1) doppler observables in the ODE:

$$F_1 = C_2\, f_{T_0} - \frac{\left[\phi\left(t_{3_e}\right) - \phi\left(t_{3_s}\right)\right]}{T_c} \qquad \text{Hz} \qquad (13\text{–}40)$$

which is not the equation that we want. The mechanical derivation of this equation substituted $C_2\, f_{T_0}$ for the doppler reference frequency $f_{\text{REF}}(t_3)$. Eq. (13–40) is equivalent to the definition (13–29) for one-way doppler (F_1) observables calculated prior to implementation of the NSP. After the NSP is implemented, we want the doppler reference frequency for F_1 observables to be zero. The desired equation for calculating the observed values of one-way doppler (F_1) observables in the ODE after the NSP is implemented is Eq. (13–40) minus the term $C_2\, f_{T_0}$. The resulting equation is Eq. (13–38).

Hence, after the NSP is implemented, observed values of one-way doppler (F_1) observables and ramped two-way (F_2) or three-way (F_3) doppler observables can be calculated in the ODE from Eq. (13–38). These observables are the negative of the average received frequency during the count interval T_c.

From Eq. (13–38), the definition of these observables calculated in the ODE is given by:

$$F_1,\ \text{ramped}\ F_{2,3} = -\frac{1}{T_c}\int_{t_{3_s}(\mathrm{ST})_R}^{t_{3_e}(\mathrm{ST})_R} f_R(t_3)\,dt_3 \qquad \text{Hz} \qquad (13–41)$$

This equation is the same as the definition (13–29) with $C_2\, f_{T_0}$ set to zero and the definition (13–33) with $f_{REF}(t_3)$ set to zero.

13.3.2 COMPUTED VALUES OF DOPPLER OBSERVABLES, MEDIA CORRECTIONS, AND PARTIAL DERIVATIVES

13.3.2.1 Unramped Two-Way (F_2) and Three-Way (F_3) Doppler Observables

These data types are obtained from receivers older than Block 5 receivers. For Block 5 receivers, these round-trip observables have been replaced with ramped two-way (F_2) and three-way (F_3) doppler observables (see Section 13.3.2.2). The Network Simplification Program will not be applied to unramped F_2 and F_3 observables.

The definition of unramped two-way (F_2) and three-way (F_3) doppler observables is given by Eq. (13–31). During an interval dt_1 of station time ST at the transmitting electronics at the transmitting station on Earth, dn cycles of the constant transmitter frequency $f_T(t_1)$ are transmitted. During the corresponding reception interval dt_3 in station time ST at the receiving electronics at the receiving station on Earth, $M_2\, dn$ cycles are received, where M_2 is the spacecraft transponder turnaround ratio. The ratio of the received frequency in cycles per second of station time ST at the receiving electronics to the transmitted frequency in cycles per second of station time ST at the transmitting electronics is given by:

$$\frac{f_R}{f_T} = \frac{M_2\, dn}{dt_3}\frac{dt_1}{dn} = M_2\frac{dt_1}{dt_3} \qquad (13–42)$$

and the received frequency in cycles per second of station time ST at the receiving electronics at the receiving station on Earth is given by:

$$f_{\mathrm{R}}(t_3) = M_2\, f_{\mathrm{T}}(t_1)\, \frac{dt_1}{dt_3} \qquad \mathrm{Hz} \qquad (13\text{–}43)$$

Substituting Eq. (13–43) into Eq. (13–31) gives:

$$\text{unramped } F_{2,3} = \frac{M_2 f_{\mathrm{T}}(t_1)}{T_{\mathrm{c}}} \left[\int_{t_{3_{\mathrm{s}}}(\mathrm{ST})_{\mathrm{R}}}^{t_{3_{\mathrm{e}}}(\mathrm{ST})_{\mathrm{R}}} dt_3 - \int_{t_{1_{\mathrm{s}}}(\mathrm{ST})_{\mathrm{T}}}^{t_{1_{\mathrm{e}}}(\mathrm{ST})_{\mathrm{T}}} dt_1 \right] \qquad \mathrm{Hz} \qquad (13\text{–}44)$$

The reception interval T_{c} at the receiving station on Earth starts at the epoch $t_{3_{\mathrm{s}}}(\mathrm{ST})_{\mathrm{R}}$ in station time ST at the receiving electronics and ends at the epoch $t_{3_{\mathrm{e}}}(\mathrm{ST})_{\mathrm{R}}$. These epochs are calculated from Eqs. (13–24) and (13–25) as functions of the data time tag TT and the count interval T_{c}. The corresponding transmission interval T_{c}' at the transmitting station on Earth starts at the epoch $t_{1_{\mathrm{s}}}(\mathrm{ST})_{\mathrm{T}}$ in station time ST at the transmitting electronics and ends at the epoch $t_{1_{\mathrm{e}}}(\mathrm{ST})_{\mathrm{T}}$. Signals transmitted at the epochs $t_{1_{\mathrm{s}}}(\mathrm{ST})_{\mathrm{T}}$ and $t_{1_{\mathrm{e}}}(\mathrm{ST})_{\mathrm{T}}$ at the transmitting electronics at the transmitting station on Earth are received at the epochs $t_{3_{\mathrm{s}}}(\mathrm{ST})_{\mathrm{R}}$ and $t_{3_{\mathrm{e}}}(\mathrm{ST})_{\mathrm{R}}$ at the receiving electronics at the receiving station on Earth, respectively. Evaluating Eq. (13–44) gives:

$$\text{unramped } F_{2,3} = \frac{M_2 f_{\mathrm{T}}(t_1)}{T_{\mathrm{c}}} \left\{ \left[t_{3_{\mathrm{e}}}(\mathrm{ST})_{\mathrm{R}} - t_{3_{\mathrm{s}}}(\mathrm{ST})_{\mathrm{R}} \right] - \left[t_{1_{\mathrm{e}}}(\mathrm{ST})_{\mathrm{T}} - t_{1_{\mathrm{s}}}(\mathrm{ST})_{\mathrm{T}} \right] \right\} \qquad \mathrm{Hz} \qquad (13\text{–}45)$$

Reordering terms gives:

$$\text{unramped } F_{2,3} = \frac{M_2 f_{\mathrm{T}}(t_1)}{T_{\mathrm{c}}} \left\{ \left[t_{3_{\mathrm{e}}}(\mathrm{ST})_{\mathrm{R}} - t_{1_{\mathrm{e}}}(\mathrm{ST})_{\mathrm{T}} \right] - \left[t_{3_{\mathrm{s}}}(\mathrm{ST})_{\mathrm{R}} - t_{1_{\mathrm{s}}}(\mathrm{ST})_{\mathrm{T}} \right] \right\} \qquad \mathrm{Hz} \qquad (13\text{–}46)$$

The definition of the precision round-trip light time ρ is given by Eq. (11–5). Using this definition, Eq. (13–46) can be expressed as:

$$\text{unramped } F_{2,3} = \frac{M_2\, f_T(t_1)}{T_c}\left(\rho_e - \rho_s\right) \qquad \text{Hz} \qquad (13\text{–}47)$$

where ρ_e and ρ_s are precision round-trip light times with reception times at the receiving electronics at the receiving station on Earth equal to $t_{3_e}(\mathrm{ST})_R$ and $t_{3_s}(\mathrm{ST})_R$, respectively. Eq. (13–47) is used to calculate the computed values of unramped two-way (F_2) and three-way (F_3) doppler observables. Each computed observable requires two round-trip spacecraft light-time solutions with reception times equal to $t_{3_e}(\mathrm{ST})_R$ and $t_{3_s}(\mathrm{ST})_R$, respectively, at the receiving electronics at the receiving station on Earth. These light-time solutions are calculated as described in Section 8.3.6. Given these light-time solutions, the precision round-trip light times ρ_e and ρ_s are calculated from Eq. (11–7) as described in Section 11.3.2. The spacecraft transponder turnaround ratio M_2 and the constant transmitter frequency $f_T(t_1)$ at the transmitting station on Earth are obtained as described in Sections 13.2.2 and 13.2.1.

The precision round-trip light times ρ_e and ρ_s do not include corrections due to the troposphere or due to charged particles. These corrections are included in the media corrections $\Delta\rho_e$ and $\Delta\rho_s$ to ρ_e and ρ_s, respectively. These media corrections are calculated in the Regres editor from Eqs. (10–28) and (10–29) as described in Section 10.2. Given the media corrections $\Delta\rho_e$ and $\Delta\rho_s$, the corresponding media correction to the computed unramped two-way (F_2) or three-way (F_3) doppler observable is calculated in the Regres editor from the following differential of Eq. (13–47):

$$\Delta\left(\text{unramped } F_{2,3}\right) = \frac{M_2\, f_T(t_1)}{T_c}\left(\Delta\rho_e - \Delta\rho_s\right) \qquad \text{Hz} \qquad (13\text{–}48)$$

This media correction to the computed observable is placed on the Regres file in the variable CRESID as discussed in Sections 10.1 and 10.2.

The partial derivatives of computed values of unramped two-way (F_2) and three-way (F_3) doppler observables with respect to solve-for and consider parameters **q** are calculated from the following partial derivative of Eq. (13–47):

$$\frac{\partial\left(\text{unramped } F_{2,3}\right)}{\partial \mathbf{q}} = \frac{M_2 f_{\mathrm{T}}(t_1)}{T_{\mathrm{c}}}\left(\frac{\partial \rho_{\mathrm{e}}}{\partial \mathbf{q}} - \frac{\partial \rho_{\mathrm{s}}}{\partial \mathbf{q}}\right) \tag{13–49}$$

The partial derivatives of the precision round-trip light times ρ_e and ρ_s at the end and start of the doppler count interval T_c with respect to the solve-for and consider parameter vector **q** are calculated from the formulation given in Section 12.5.1 as described in that section.

13.3.2.2 Ramped Two-Way (F_2) and Three-Way (F_3) Doppler Observables

The formulation for calculating the computed value of a ramped two-way (F_2) or three-way (F_3) doppler observable, the correction to the computed value of the observable due to media corrections, and the partial derivatives of the computed observable with respect to solve-for and consider parameters is given in Subsection 13.3.2.2.1. The equation for the computed value of a ramped F_2 or F_3 doppler observable contains the integral of the transmitted frequency over the transmission interval $T_c{}'$ and the integral of the doppler reference frequency over the reception interval T_c. If this latter integral is ramped, it can only be evaluated using ramp tables, as described in Subsection 13.3.2.2.2. The former integral can be evaluated using ramp tables as described in Subsection 13.3.2.2.2, or it can be evaluated using phase tables as described in Subsection 13.3.2.2.3.

<u>13.3.2.2.1</u> <u>Formulation</u>

Ramped two-way (F_2) and three-way (F_3) doppler observables have been and are obtained at receivers older than Block 5 receivers, are obtained from Block 5 receivers prior to implementation of the Network Simplification Program (NSP), and will be obtained from Block 5 receivers after the NSP is implemented. All three cases are discussed in this section.

The definition of ramped two-way (F_2) and three-way (F_3) doppler observables is given by Eq. (13–33). For receivers older than Block 5 receivers, the doppler reference frequency $f_{\mathrm{REF}}(t_3)$ can be constant or ramped. For Block 5 receivers prior to implementation of the NSP, $f_{\mathrm{REF}}(t_3)$ is a fictitious constant frequency. For Block 5 receivers after the NSP is implemented, $f_{\mathrm{REF}}(t_3)$ will be zero. The simulation synthesizer flag on the data record of the OD file specifies whether $f_{\mathrm{REF}}(t_3)$ is constant or ramped. Constant values specified at level 0, 1, or 2 are obtained from the data record of the OD file. If $f_{\mathrm{REF}}(t_3)$ is ramped, the ramp records of the OD file contain ramp tables for the transmitter frequency $f_{\mathrm{T}}(t_3)$ or the reference oscillator frequency $f_{\mathrm{q}}(t_3)$ at the receiving station on Earth. Constant or ramped values of $f_{\mathrm{REF}}(t_3)$ are calculated as described in Section 13.2.4.

In Eq. (13–33), the doppler reference frequency $f_{\mathrm{REF}}(t_3)$ is given by Eq. (13–9), and the received frequency is given by Eq. (13–43). Substituting these equations into Eq. (13–33) gives:

$$\text{ramped } F_{2,3} = \frac{M_{2_{\mathrm{R}}}}{T_{\mathrm{c}}} \int_{t_{3_{\mathrm{s}}}(\mathrm{ST})_{\mathrm{R}}}^{t_{3_{\mathrm{e}}}(\mathrm{ST})_{\mathrm{R}}} f_{\mathrm{T}}(t_3)\, dt_3 - \frac{M_2}{T_{\mathrm{c}}} \int_{t_{1_{\mathrm{s}}}(\mathrm{ST})_{\mathrm{T}}}^{t_{1_{\mathrm{e}}}(\mathrm{ST})_{\mathrm{T}}} f_{\mathrm{T}}(t_1)\, dt_1 \quad \mathrm{Hz} \qquad (13\text{–}50)$$

where the epochs at the start and end of the reception interval T_{c} and the corresponding transmission interval T_{c}' are described after Eq. (13–44). The transmitter frequency $f_{\mathrm{T}}(t_1)$ at the transmitting station on Earth is ramped. Its value can be obtained from the ramp table for the transmitting station. The frequency and the accumulated phase of the transmitted signal can be obtained by interpolating the phase table for the transmitting station. These tables are obtained from the OD file. The spacecraft transponder turnaround ratio M_2 is calculated as described in Section 13.2.2. The spacecraft turnaround ratio $M_{2_{\mathrm{R}}}$ built into the electronics at the receiving station on Earth is calculated as described in Section 13.2.4.

Eq. (13–50) is used to calculate the computed values of ramped two-way (F_2) and three-way (F_3) doppler observables. Each computed observable requires two round-trip spacecraft light-time solutions with reception times equal to $t_{3_e}(\text{ST})_R$ and $t_{3_s}(\text{ST})_R$, respectively, at the receiving electronics at the receiving station on Earth. These light-time solutions are calculated as described in Section 8.3.6. Given these light-time solutions, the precision round-trip light times ρ_e and ρ_s are calculated from Eq. (11–7) as described in Section 11.3.2. The integrals in Eq. (13–50) are evaluated from ramp tables or phase tables as described in Sections 13.3.2.2.2 and 13.3.2.2.3, respectively. Evaluation of each of these integrals requires the precision width of the interval of integration. The precision width of the reception interval at the receiving electronics at the receiving station on Earth is the count interval T_c. The precision width of the transmission interval at the transmitting electronics at the transmitting station on Earth is calculated from:

$$T_c{}' = T_c - \left(\rho_e - \rho_s\right) \qquad \text{s} \qquad (13\text{–}51)$$

Evaluation of the integrals in Eq. (13–50) also requires the epochs $t_{3_s}(\text{ST})_R$ and $t_{3_e}(\text{ST})_R$ at the start and end of the reception interval and the epochs $t_{1_s}(\text{ST})_T$ and $t_{1_e}(\text{ST})_T$ at the start and end of the transmission interval. The former epochs are calculated from Eqs. (13–25) and (13–24). The latter epochs can be calculated two different ways. The least accurate way is to start with the transmission times $t_{1_s}(\text{ST})$ and $t_{1_e}(\text{ST})$ at the tracking station location from the light-time solutions at the start and end of the count interval. They are converted to the corresponding transmission times $t_{1_s}(\text{ST})_T$ and $t_{1_e}(\text{ST})_T$ at the transmitting electronics using Eq. (11–3) and the up-leg delay τ_U at the transmitting station on Earth. However, a more accurate way is to calculate $t_{1_s}(\text{ST})_T$ and $t_{1_e}(\text{ST})_T$ from:

$$t_{1_s}(\text{ST})_T = t_{3_s}(\text{ST})_R - \rho_s \qquad \text{s} \qquad (13\text{–}52)$$

$$t_{1_e}(\text{ST})_T = t_{3_e}(\text{ST})_R - \rho_e \qquad \text{s} \qquad (13\text{–}53)$$

This method is more accurate because ρ_s and ρ_e contain some small terms which are not calculated in the spacecraft light-time solution.

The following two sections give the algorithms for evaluating the integrals in Eq. (13–50).

If the doppler reference frequency $f_{\mathrm{REF}}\left(t_3\right)$ at the receiving station on Earth is constant, Eq. (13–50) reduces to:

$$\text{ramped } F_{2,3} = f_{\mathrm{REF}}\left(t_3\right) - \frac{M_2}{T_c} \int_{t_{1_s}(\mathrm{ST})_T}^{t_{1_e}(\mathrm{ST})_T} f_T\left(t_1\right) dt_1 \qquad \mathrm{Hz} \qquad (13\text{–}54)$$

The constant value of $f_{\mathrm{REF}}\left(t_3\right)$ is calculated as described in Section 13.2.4. For Block 5 receivers after the NSP is implemented, $f_{\mathrm{REF}}\left(t_3\right)$ will be zero. For this case, Eq. (13–54) is equal to Eq. (13–38) since the number of cycles received during the reception interval is equal to the number of cycles transmitted during the transmission interval multiplied by the spacecraft turnaround ratio M_2.

The precision round-trip light times ρ_e and ρ_s do not include corrections due to the troposphere or due to charged particles. These corrections are included in the media corrections $\Delta\rho_e$ and $\Delta\rho_s$ to ρ_e and ρ_s, respectively. These media corrections are calculated in the Regres editor from Eqs. (10–28) and (10–29) as described in Section 10.2. Since the reception times at the end and start of the reception interval T_c are fixed, the media corrections $\Delta\rho_e$ and $\Delta\rho_s$ are the negatives of the corresponding changes in the transmission times at the transmitting station on Earth:

$$\Delta\rho_e = -\,\Delta t_{1_e}(\mathrm{ST})_T \qquad \mathrm{s} \qquad (13\text{–}55)$$

$$\Delta\rho_s = -\,\Delta t_{1_s}(\mathrm{ST})_T \qquad \mathrm{s} \qquad (13\text{–}56)$$

From Eq. (13–50), the media correction to a computed ramped two-way (F_2) or three-way (F_3) doppler observable is given by:

$$\Delta\left(\text{ramped } F_{2,3}\right) = -\frac{M_2}{T_c}\left[f_T\left(t_{1_e}\right)\Delta t_{1_e}(\text{ST})_T - f_T\left(t_{1_s}\right)\Delta t_{1_s}(\text{ST})_T \right] \quad \text{Hz} \qquad (13–57)$$

Substituting Eqs. (13–55) and (13–56) into Eq. (13–57) gives:

$$\Delta\left(\text{ramped } F_{2,3}\right) = \frac{M_2}{T_c}\left[f_T\left(t_{1_e}\right)\Delta\rho_e - f_T\left(t_{1_s}\right)\Delta\rho_s \right] \quad \text{Hz} \qquad (13–58)$$

This equation is used to calculate media corrections for computed ramped two-way (F_2) and three-way (F_3) doppler observables in the Regres editor. The transmitter frequencies $f_T\left(t_{1_s}\right)$ at the start and $f_T\left(t_{1_e}\right)$ at the end of the transmission interval at the transmitting station on Earth are obtained in evaluating the second integral in Eq. (13–50) using the algorithm of Section 13.3.2.2.2 or 13.3.2.2.3. Program Regres writes these frequencies onto the Regres file, which is read by the Regres editor. These transmitter frequencies are used directly in Eq. (13–58) and indirectly in evaluating the media corrections $\Delta\rho_e$ and $\Delta\rho_s$ using Eqs. (13–19) and (13–20). The Regres editor evaluates the media correction (13–58) to the computed observable and places it on the Regres file in the variable CRESID as discussed in Sections 10.1 and 10.2.

Replacing corrections (Δ) in Eqs. (13–55) to (13–58) with partial derivatives with respect to the solve-for and consider parameter vector **q**, the partial derivatives of the computed values of ramped two-way (F_2) and three-way (F_3) doppler observables with respect to solve-for and consider parameters **q** are calculated from:

$$\frac{\partial\left(\text{ramped } F_{2,3}\right)}{\partial \mathbf{q}} = \frac{M_2}{T_c}\left[f_T\left(t_{1_e}\right)\frac{\partial \rho_e}{\partial \mathbf{q}} - f_T\left(t_{1_s}\right)\frac{\partial \rho_s}{\partial \mathbf{q}} \right] \qquad (13–59)$$

The partial derivatives of the precision round-trip light times ρ_e and ρ_s at the end and start of the doppler count interval T_c with respect to the solve-for and consider parameter vector **q** are calculated from the formulation given in Section 12.5.1 as described in that section.

13.3.2.2.2 Evaluating Integrals Using Ramp Tables

If the doppler reference frequency (given by Eq. 13–9) at the receiving station on Earth is ramped, the integral of the ramped transmitter frequency over the count interval T_c at the receiving station (the first term of Eq. 13–50) is calculated using the ramp table for the receiving station. The integral of the ramped transmitter frequency over the transmission interval T_c' at the transmitting station (the second term of Eq. 13–50) can be calculated from the ramp table or the phase table for the transmitting station. This section describes how the two integrals in Eq. (13–50) are evaluated using the ramp tables for the receiving and transmitting stations.

The following algorithm can be used to evaluate either integral in Eq. (13–50). Let t_s denote the start time of the interval of integration. It is $t_{3_s}(\mathrm{ST})_R$ for the first integral and $t_{1_s}(\mathrm{ST})_T$ for the second integral of Eq. (13–50). Let t_e denote the end time of the interval of integration. It is $t_{3_e}(\mathrm{ST})_R$ for the first integral and $t_{1_e}(\mathrm{ST})_T$ for the second integral of Eq. (13–50). These four epochs are calculated from Eqs. (13–24), (13–25), (13–52), and (13–53) as discussed after Eq. (13–51). Let W denote the precision width of the interval of integration. It is T_c for the reception interval and T_c' given by Eq. (13–51) for the transmission interval. The interval of integration is covered by n ramps, where n can be as few as one. Each ramp is specified by the start time t_o and end time t_f of the ramp in station time ST at the tracking station (integer seconds), the value f_o of the ramped transmitter frequency $f_T(t)$ at the start time t_o of the ramp, and the constant time derivative (the ramp rate) $\dot{f}$ of $f_T(t)$ which applies from t_o to t_f. The start time t_o for the first ramp is before the start time t_s of the interval of integration. The end time t_f of the last ramp is after the end time t_e of the interval of integration. The following steps produce the value of either integral in Eq. (13–50):

1. Change the start time of the first ramp from t_o to the start time t_s of the interval of integration.

2. Change the transmitter frequency at the start of the first ramp to:

$$f_o(\text{at } t_s) = f_o(\text{at } t_o) + \dot{f}\,(t_s - t_o) \qquad \text{Hz} \qquad (13\text{–}60)$$

3. If the interval of integration W contains two or more ramps, calculate the width of each ramp i except the last ramp from:

$$W_i = t_f - t_o \qquad \text{s} \qquad (13\text{–}61)$$

where the recalculated value of t_o for the first ramp is obtained from Step 1.

4. If W contains two or more ramps, calculate the precision width of the last ramp from:

$$W_n = W - \sum_{i=1}^{n-1} W_i \qquad \text{s} \qquad (13\text{–}62)$$

where W is the precision width of the interval of integration, obtained as described above. If W contains one ramp only, its precision width is:

$$W_n(n=1) = W_1 = W \qquad \text{s} \qquad (13\text{–}63)$$

5. Calculate the average transmitter frequency f_i for each ramp:

$$f_i = f_o + \tfrac{1}{2}\,\dot{f}\,W_i \qquad \text{Hz} \qquad (13\text{–}64)$$

6. Evaluate the integral of the transmitter frequency over the reception or transmission interval W from:

$$\int_{t_s}^{t_e} f_T(t)\,dt = \sum_{i=1}^{n} f_i\,W_i \qquad \text{cycles} \qquad (13\text{–}65)$$

7. In addition to the integral (13–65), program Regres and the Regres editor need the values of the transmitter frequency at the start t_s and end t_e of the interval of integration. The value of $f_T(t)$ at t_s is obtained from Step 2 using Eq. (13–60). The value of $f_T(t)$ at t_e is calculated from:

$$f_e = f_o + \dot{f}\, W_n \qquad \text{Hz} \qquad (13\text{–}66)$$

where f_o and $\dot{f}$ are the values for the last ramp and W_n is the width of the last ramp calculated from Eq. (13–62) or (13–63).

13.3.2.2.3 Evaluating Integrals Using Phase Tables

The integral of the ramped transmitter frequency over the transmission interval $T_c{}'$ at the transmitting station (in the second term of Eq. 13–50 or 13–54) can be calculated from the ramp table or the phase table for the transmitting station. This section describes how this integral is evaluated using the phase table for the transmitting station on Earth.

The integral in the second term of Eq. (13–50) or (13–54) can be expressed as:

$$\int_{t_{1_s}(\mathrm{ST})_T}^{t_{1_e}(\mathrm{ST})_T} f_T(t_1)\, dt_1 = \phi\left[t_{1_e}(\mathrm{ST})_T\right] - \phi\left[t_{1_s}(\mathrm{ST})_T\right] \qquad \text{cycles} \qquad (13\text{–}67)$$

The epochs $t_{1_e}(\mathrm{ST})_T$ and $t_{1_s}(\mathrm{ST})_T$ are the end and start, respectively, of the transmission interval at the transmitting electronics at the transmitting station on Earth, measured in station time ST at the transmitting station. The terms on the right-hand side of Eq. (13–67) are the corresponding phases of the transmitted signal at the transmitting electronics at these epochs. The remainder of this section describes how the right-hand side of Eq. (13–67) is evaluated using the phase table for the transmitting station on Earth.

The epochs $t_{1_e}(\mathrm{ST})_T$ and $t_{1_s}(\mathrm{ST})_T$ are calculated from Eqs. (13–24), (13–25), (13–52), and (13–53). The phase table for the transmitting station on Earth is interpolated at these two epochs (the end and start of the transmission interval) as described in Section 13.2.7. Each interpolation produces three phase-time pairs on the same ramp: ϕ_1 at t_1, ϕ_2 at t_2, and ϕ_3 at t_3. The interpolation time t is between t_1 and t_3. Substituting the three phases and the corresponding tabular times into Eqs. (13–11) to (13–14) gives the frequency f_2 of the transmitted signal at the tabular time t_2 and the ramp rate $\dot{f}$ (which is constant between t_1 and t_3). The epoch t_2 obtained during the interpolation at the end of the transmission interval will be denoted as T_e. The epoch t_2 obtained during the interpolation at the start of the transmission interval will be denoted as T_s.

The variable Δt is defined by Eq. (13–15). It is the interpolation time t minus the corresponding tabular time t_2. The value of Δt at the start of the transmission interval is calculated from:

$$\Delta t_s = t_{1_s}(\mathrm{ST})_T - T_s \qquad \mathrm{s} \qquad (13\text{–}68)$$

The variable Δt at the end of the transmission interval is defined by Eq. (13–68) with each subscript s replaced with the subscript e. However, it is calculated from:

$$\Delta t_e = T_c' - \Delta T + \Delta t_s \qquad \mathrm{s} \qquad (13\text{–}69)$$

where the precision width T_c' of the transmission interval is calculated from Eq. (13–51), the variable ΔT is calculated from:

$$\Delta T = T_e - T_s \qquad \mathrm{s} \qquad (13\text{–}70)$$

and Δt_s is given by Eq. (13–68). Eq. (13–69) places the roundoff error in Δt_s into Δt_e. If Eq. (13–69) is solved for T_c', the roundoff errors in Δt_e and Δt_s cancel and the precision width of the transmission interval is preserved.

The parameter $\Delta\phi(\Delta t)$ is defined by Eq. (13–16). It is the phase of the transmitted signal at the interpolation time t minus the tabular phase ϕ_2 at the tabular time t_2. Given f_2 and $\dot{f}$ obtained from the interpolation at the end of the transmission interval, and Δt_e calculated from Eq. (13–69), the phase difference

$$\Delta\phi(\Delta t_e) \qquad \text{cycles} \qquad (13\text{–}71)$$

is calculated from Eq. (13–17). This is the phase of the transmitted signal at the end $t_{1_e}(\mathrm{ST})_T$ of the transmission interval minus the tabular phase ϕ_2 at the tabular time t_2. Similarly, given f_2 and $\dot{f}$ obtained from the interpolation at the start of the transmission interval, and Δt_s calculated from Eq. (13–68), the phase difference

$$\Delta\phi(\Delta t_s) \qquad \text{cycles} \qquad (13\text{–}72)$$

is calculated from Eq. (13–17). This is the phase of the transmitted signal at the start $t_{1_s}(\mathrm{ST})_T$ of the transmission interval minus the tabular phase ϕ_2 at the tabular time t_2. The variables f_2, $\dot{f}$, and Δt at the end and start of the transmission interval are also used in Eq. (13–18) to calculate values of the transmitted frequency $f_T(t)$ at the end and start of the transmission interval:

$$f_T\left[t_{1_e}(\mathrm{ST})_T\right],\ f_T\left[t_{1_s}(\mathrm{ST})_T\right] \qquad \text{Hz} \qquad (13\text{–}73)$$

Given the phase differences $\Delta\phi(\Delta t_e)$, $\Delta\phi(\Delta t_s)$, and the tabular phases $\phi(T_e)$ and $\phi(T_s)$ at the tabular times t_2 at the end and start of the transmission interval, the phase difference on the right-hand side of Eq. (13–67) is calculated from:

$$\phi\left[t_{1_e}(\mathrm{ST})_T\right] - \phi\left[t_{1_s}(\mathrm{ST})_T\right] = \Delta\phi(\Delta t_e) - \Delta\phi(\Delta t_s) + \left[\phi(T_e) - \phi(T_s)\right] \qquad \text{cycles} \qquad (13\text{–}74)$$

The difference of the tabular phases should be calculated in quadruple precision and then rounded to double precision.

13.3.2.3 One-Way (F_1) Doppler Observables

There are two versions of the formulation used to calculate the computed values of one-way (F_1) doppler observables. The original version of the F_1 formulation is used for receivers older than Block 5 receivers and also for Block 5 receivers prior to implementation of the Network Simplification Program (NSP). The newer version of the F_1 formulation is used for Block 5 receivers after implementation of the NSP. A flag on the OD file indicates whether the observed values of the observables were generated before or after implementation of the NSP.

The definition of one-way (F_1) doppler observables obtained before implementation of the NSP is given by Eq. (13–29). After implementation of the NSP, the definition of F_1 observables changes to Eq. (13–41), which is the second term of Eq. (13–29). Note that the computed value of an F_1 observable calculated from the newer formulation (after NSP) is equal to the value computed from the original formulation (prior to NSP) minus the constant frequency $C_2 f_{T_0}$, which is the nominal value of the transmitted frequency at the spacecraft. The newer formulation (after NSP) for the computed values of one-way (F_1) doppler observables will be developed first. Then, $C_2 f_{T_0}$ will be added to give the older (prior to NSP) formulation.

As stated above, the definition of one-way (F_1) doppler observables obtained after implementation of the NSP is given by Eq. (13–41). The ratio of the received frequency at the receiving electronics at the receiving station on Earth in cycles per second of station time ST at the receiving station to the transmitted frequency at the spacecraft in cycles per second of International Atomic Time TAI at the spacecraft is given by:

$$\frac{f_R}{f_T} = \frac{dn}{dt_3(\mathrm{ST})_R} \frac{dt_2(\mathrm{TAI})}{dn} = \frac{dt_2(\mathrm{TAI})}{dt_3(\mathrm{ST})_R} \tag{13–75}$$

where dn is an infinitesimal number of cycles transmitted and received. If the spacecraft were placed at mean sea level on Earth, the spacecraft atomic clock would run at the same rate as International Atomic Time on Earth (see Sections

11.4, 11.4.1, and 11.4.2). The transmitted frequency at the spacecraft in cycles per second of atomic time TAI at the spacecraft is given by Eqs. (13–7) and (13–8) as explained in Section 13.2.3. Substituting Eqs. (13–75), (13–7), and (13–8) into Eq. (13–41) gives:

$$F_1(\text{after NSP}) = -\frac{C_2}{T_c}\int_{t_{2_s}(\text{TAI})}^{t_{2_e}(\text{TAI})}\left[f_{T_0} + \Delta f_{T_0} + f_{T_1}\left(t_2 - t_0\right) + f_{T_2}\left(t_2 - t_0\right)^2 \right] dt_2(\text{TAI}) \quad \text{Hz} \qquad (13\text{–}76)$$

The reception interval at the receiving station on Earth is the count interval T_c. The epoch $t_{3_s}(\text{ST})_R$ at the start of the count interval and the epoch $t_{3_e}(\text{ST})_R$ at the end of the count interval are calculated from Eqs. (13–25) and (13–24), respectively. The transmission interval $T_c{}'$ at the spacecraft in seconds of International Atomic Time TAI at the spacecraft is calculated from:

$$T_c{}' = T_c - \left(\hat{\rho}_{1_e} - \hat{\rho}_{1_s}\right) \quad \text{s} \qquad (13\text{–}77)$$

The precision one-way light times $\hat{\rho}_{1_e}$ and $\hat{\rho}_{1_s}$ are defined by Eq. (11–8). They have reception times equal to the end $t_{3_e}(\text{ST})_R$ and start $t_{3_s}(\text{ST})_R$ of the reception interval at the receiving electronics at the receiving station on Earth and transmission times equal to the end $t_{2_e}(\text{TAI})$ and start $t_{2_s}(\text{TAI})$ of the transmission interval at the spacecraft.

The precision one-way light times $\hat{\rho}_{1_e}$ and $\hat{\rho}_{1_s}$ defined by Eq. (11–8) cannot be computed directly because we do not have a model for the time difference $(\text{ET} - \text{TAI})_{t_2}$ at the spacecraft. The differenced one-way light time $\hat{\rho}_{1_e} - \hat{\rho}_{1_s}$ in Eq. (13–77) is calculated from Eq. (11–11), where the precision one-way light times ρ_{1_e} and ρ_{1_s} are defined by Eq. (11–9) and Δ, which is defined by Eq. (11–12), is the change in the time difference $(\text{ET} - \text{TAI})_{t_2}$ that occurs during the transmission interval $T_c{}'$ at the spacecraft. The precision one-way light times ρ_{1_e} and ρ_{1_s} are calculated from Eq. (11–41) using quantities calculated in and after the one-way light-time solutions at the end and start of the count interval.

The parameter Δ is calculated from Eqs. (11–15) to (11–39) in Sections 11.4.2 and 11.4.3. This algorithm uses quantities calculated at the transmission time t_2 in the light-time solutions at the end and start of the count interval.

In Eq. (13–76), the parameters Δf_{T_0}, f_{T_1}, and f_{T_2} are the coefficients of the quadratic offset of the S-band value of the spacecraft transmitter frequency $f_{S/C}$ from its nominal value f_{T_0} (see Eq. 13–8). The upper and lower limits of the interval of integration in Eq. (13–76) are only required to evaluate the terms containing the coefficients f_{T_1} and f_{T_2}. Since these terms are small, the limits of integration can be replaced with the corresponding values in coordinate time ET:

$$t_{2_e}(\mathrm{ET}),\ t_{2_s}(\mathrm{ET}) \qquad \mathrm{s} \qquad (13\text{–}78)$$

These epochs can be calculated from:

$$t_{2_e}(\mathrm{ET}) = t_{3_e}(\mathrm{ST})_\mathrm{R} - \rho_{1_e} \qquad \mathrm{s} \qquad (13\text{–}79)$$

$$t_{2_s}(\mathrm{ET}) = t_{3_s}(\mathrm{ST})_\mathrm{R} - \rho_{1_s} \qquad \mathrm{s} \qquad (13\text{–}80)$$

where $t_{3_e}(\mathrm{ST})_\mathrm{R}$ and $t_{3_s}(\mathrm{ST})_\mathrm{R}$ are given by Eqs. (13–24) and (13–25). We will need the average of the ET values of the epochs at the start and end of the transmission interval at the spacecraft:

$$t_{2_m} = \frac{t_{2_e}(\mathrm{ET}) + t_{2_s}(\mathrm{ET})}{2} \qquad \mathrm{s} \qquad (13\text{–}81)$$

Evaluating the integral in Eq. (13–76) using the above approximation gives:

$$F_1\,(\text{after NSP}) =$$

$$-\,C_2 \left\{ f_{T_0} + \Delta f_{T_0} + f_{T_1}\left(t_{2_m} - t_0\right) + f_{T_2}\left[\left(t_{2_m} - t_0\right)^2 + \frac{1}{12}\left(T_c{}'\right)^2\right] \right\} \frac{T_c{}'}{T_c} \qquad \mathrm{Hz} \qquad (13\text{–}82)$$

where t_{2_m} is given by Eqs. (13–79) to (13–81). The quadratic coefficients Δf_{T_0}, f_{T_1}, and f_{T_2} are specified by time block with start time t_0. The coefficients selected are those for the time block that contains t_{2_m}. The transmission interval $T_c^{\prime}$ at the spacecraft is calculated from Eq. (13–77). From this equation, $T_c^{\prime} / T_c$ in Eq. (13–82) is given by:

$$\frac{T_c^{\prime}}{T_c} = 1 - \frac{\hat{\rho}_{1_e} - \hat{\rho}_{1_s}}{T_c} \tag{13–83}$$

The differenced one-way light time $\hat{\rho}_{1_e} - \hat{\rho}_{1_s}$ in Eqs. (13–77) and (13–83) is calculated from the formulation of Section 11.4 as described above.

As discussed in the second paragraph of this section, the computed value of a one-way doppler (F_1) observable prior to implementation of the NSP is given by Eq. (13–82) plus the constant frequency $C_2 f_{T_0}$. Using Eq. (13–83), the resulting equation is given by:

$$\begin{aligned} F_1\left(\text{before NSP}\right) = {} & C_2 f_{T_0} \frac{\left(\hat{\rho}_{1_e} - \hat{\rho}_{1_s}\right)}{T_c} \\ & - C_2 \left\{ \Delta f_{T_0} + f_{T_1}\left(t_{2_m} - t_0\right) + f_{T_2}\left[\left(t_{2_m} - t_0\right)^2 + \frac{1}{12}\left(T_c^{\prime}\right)^2\right]\right\} \frac{T_c^{\prime}}{T_c} \\ & \text{Hz} \end{aligned} \tag{13–84}$$

Eqs. (13–82) and (13–84) contain $T_c^{\prime}$ calculated from Eq. (13–77) and $T_c^{\prime} / T_c$ calculated from Eq. (13–83). These equations contain the differenced one-way light time $\hat{\rho}_{1_e} - \hat{\rho}_{1_s}$, which is calculated from Eq. (11–11). In this equation, the precision one-way light times ρ_{1_e} and ρ_{1_s}, which are calculated from Eq. (11–41), do not contain corrections due to the troposphere and charged particles. These corrections are included in the media corrections $\Delta\rho_{1_e}$ and $\Delta\rho_{1_s}$ to ρ_{1_e} and ρ_{1_s}, respectively. These media corrections are calculated in the Regres editor from Eqs. (10–24) and (10–25) as described in Section 10.2. Given the media corrections $\Delta\rho_{1_e}$ and $\Delta\rho_{1_s}$, the corresponding media correction to the computed one-way (F_1) doppler observable is calculated in the Regres editor from the following differential of Eq. (13–82) or (13–84):

$$\Delta F_1 = C_2 \, f_{S/C}{}^{*} \, \frac{\left(\Delta\rho_{1_e} - \Delta\rho_{1_s}\right)}{T_c} \qquad \text{Hz} \qquad (13\text{–}85)$$

where

$$f_{S/C}{}^{*} = f_{T_0} + \Delta f_{T_0} + f_{T_1}\left(t_{2_m} - t_0\right) + f_{T_2}\left[\left(t_{2_m} - t_0\right)^2 + \frac{1}{4}\left(T_c{}'\right)^2\right] \qquad \text{Hz} \qquad (13\text{–}86)$$

In deriving Eqs. (13–85) and (13–86), the media correction $-\left(\Delta\rho_{1_e} + \Delta\rho_{1_s}\right)/2$ to t_{2_m}, which produces a negligible change to F_1, has been ignored.

The partial derivatives of computed values of one-way (F_1) doppler observables with respect to solve-for and consider parameters **q** are calculated from the following partial derivative of Eq. (13–82) or (13–84):

$$\frac{\partial F_1}{\partial \mathbf{q}} = \frac{C_2 \, f_{S/C}{}^{*}}{T_c}\left(\frac{\partial \rho_{1_e}}{\partial \mathbf{q}} - \frac{\partial \rho_{1_s}}{\partial \mathbf{q}}\right) \qquad (13\text{–}87)$$

where $f_{S/C}{}^{*}$ is given by Eq. (13–86). The partial derivatives of the precision one-way light times ρ_{1_e} and ρ_{1_s} at the end and start of the doppler count interval T_c with respect to the solve-for and consider parameter vector **q** are calculated from the formulation given in Section 12.5.2 as described in that section. Eq. (13–87) ignores the effect of the parameter vector **q** on ρ_{1_e}, ρ_{1_s}, and t_{2_m} obtained using Eqs. (13–79) to (13–81).

In addition to the partial derivatives given by Eq. (13–87), the partial derivatives of F_1 with respect to the quadratic coefficients of the offset of $f_{S/C}$ from f_{T_0} are obtained by differentiating Eq. (13–82) or (13–84):

$$\frac{\partial F_1}{\partial \Delta f_{T_0}} = -C_2 \frac{T_c{}'}{T_c} \tag{13–88}$$

$$\frac{\partial F_1}{\partial f_{T_1}} = -C_2 \left(t_{2_m} - t_0\right) \frac{T_c{}'}{T_c} \tag{13–89}$$

$$\frac{\partial F_1}{\partial f_{T_2}} = -C_2 \left[\left(t_{2_m} - t_0\right)^2 + \frac{1}{12}\left(T_c{}'\right)^2 \right] \frac{T_c{}'}{T_c} \tag{13–90}$$

13.4 TOTAL-COUNT PHASE OBSERVABLES

13.4.1 INTRODUCTION

A total-count phase observable can be obtained from the corresponding doppler observable (with the same count interval T_c) by multiplying it by T_c. This relationship applies for the observed and computed values of these data types, the correction to the computed observable due to media effects, and the partial derivative of the computed observable with respect to the parameter vector **q**. It applies for one-way doppler (F_1) and phase (P_1) and two-way and three-way doppler (F_2 and F_3) and phase (P_2 and P_3).

Total-count phase observables will be available after the Network Simplification Program (NSP) is completed for data points which have a Block 5 receiver at the receiving station on Earth and (if the transmitter is a tracking station on Earth) a Block 5 exciter at the transmitting station on Earth. The ODE will be modified to process these data types after the NSP is completed. Program Regres can already process these data types. After the NSP is implemented, round-trip F_2 and F_3 observables obtained at a station with a Block 5 receiver will be ramped, and the doppler reference frequency will be zero. One-way F_1 observables obtained at a station with a Block 5 receiver will correspond to the slightly-modified definition given in Section 13.3. The observed and computed values of these doppler observables, the correction to the computed doppler observables due to media effects, and the partial derivatives of the computed doppler observables with respect to the parameter vector **q** all include a divide by the count interval T_c. Hence, the corresponding quantities for total-count phase observables can be calculated from the corresponding doppler formulation, except that the divide by T_c must be suppressed.

Doppler observables have units of cycles per second or Hz. Since total-count phase observables are doppler observables multiplied by the count interval T_c, they have units of cycles. In addition to the difference in units, total-count phase observables have a different configuration of the count intervals than that used for doppler observables. Figure 13–1 shows the contiguous count intervals of width T_c used for six doppler observables received during a pass of data at a tracking station on Earth:

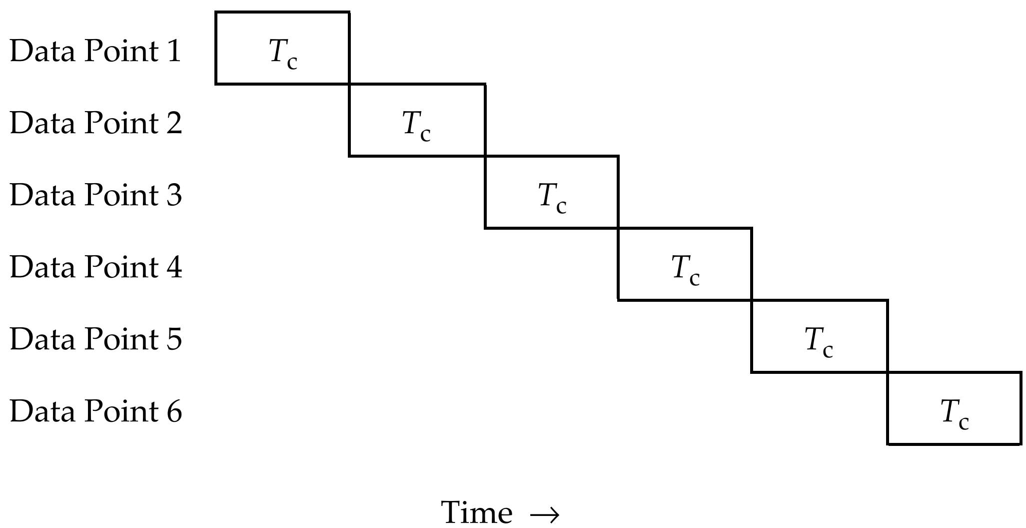

Figure 13–1 Count Intervals For Doppler Observables

Note that the end of each count interval (reception interval) is the beginning of the next interval. Figure 13–2 shows the count intervals used for six total-count phase observables received during a pass of data at a tracking station on Earth:

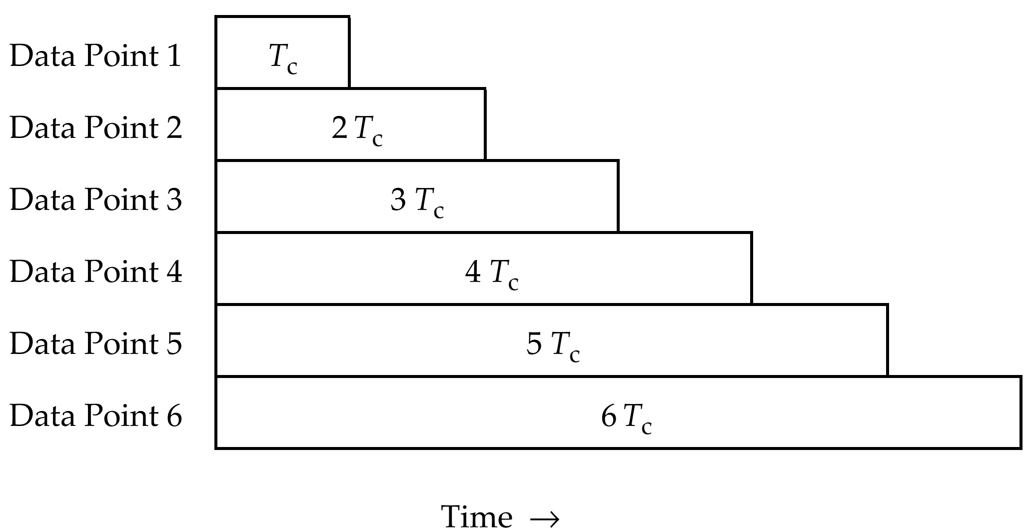

Figure 13–2 Count Intervals For Total-Count Phase Observables

Note that the start time for each count interval (reception interval) is the same epoch, which is near the start of the pass of data at the tracking station. Also note that the count intervals for data points 1 through 6 have widths of T_c, 2 T_c, 3 T_c, 4 T_c, 5 T_c, and 6 T_c, respectively, where T_c is the doppler count interval. As long as the accumulated counted phase of the received signal at the receiving station on Earth is continuous (*i.e.*, there are no cycle slips), the count intervals for successive total-count phase observables can approach the full length of the pass of data. If the spacecraft does not set at a given tracking station on Earth, then the pass of data may be several days long. If the counted phase of the received signal is discontinuous, then the start time for all count intervals after the discontinuity will have to be changed to an epoch after the discontinuity. Each discontinuity reduces the power of total-count phase data. This is discussed further in Section 13.4.2.

The weight for each data point is one divided by the square of the calculated standard deviation for the data point. Doppler data points have an input nominal standard deviation, which is modified according to the width of the count interval and the elevation angle of the spacecraft. Consider a total-count phase observable with a count interval of nT_c, where T_c is the doppler count interval. If the standard deviation for the total-count phase observable were taken to be the calculated standard deviation for the doppler data point multiplied by nT_c, it would be proportional to n and the power of the total-count phase observable would be lost. Instead, the standard deviation for total-count phase observables will be an input constant, regardless of how long the count interval grows during the pass of data at a tracking station. The standard deviation will probably be a fraction of a cycle to a few cycles of the received signal at the tracking station on Earth. The number used will vary with the band of the received signal.

13.4.2 OBSERVED VALUES OF TOTAL-COUNT PHASE OBSERVABLES

For each receiving station on Earth, the ODE must determine the intervals of time during which the accumulated phase $\phi(t_3)$ of the received signal (defined by Eq. 13–34) is continuous. Then, given the user's desired doppler count interval

T_c, the ODE can determine the number of total-count phase count intervals of duration T_c, $2\,T_c$, $3\,T_c$, $4\,T_c$, etc. that will fit into each continuous reception interval, as shown in Figure 13–2. For the remainder of Section 13.4, T_c will refer to the count interval for a total-count phase observable.

After the NSP is implemented, observed values of one-way doppler (F_1) observables and ramped two-way (F_2) or three-way (F_3) doppler observables obtained at a tracking station on Earth which has a Block 5 receiver can be calculated from Eq. (13–38). Multiplying this equation by the count interval T_c for total-count phase observables gives the following equation for calculating the observed values of one-way total-count phase (P_1) observables and ramped two-way (P_2) or three-way (P_3) total-count phase observables obtained at a tracking station on Earth which has a Block 5 receiver:

$$P_1\text{, ramped } P_{2,3} = -\left[\phi\left(t_{3_\mathrm{e}}\right) - \phi\left(t_{3_\mathrm{s}}\right)\right] \qquad \text{cycles} \qquad (13\text{–}91)$$

where $\phi\left(t_{3_\mathrm{e}}\right)$ and $\phi\left(t_{3_\mathrm{s}}\right)$ are values of the accumulated phase $\phi(t_3)$ of the received signal (defined by Eq. 13–34) at the end and start of the count interval T_c for the total-count phase observable. For total-count phase observables, the time tag is the end of the count interval. Hence, given the time tag TT and count interval T_c for a total-count phase observable, the epochs at the end and start of the count interval are calculated from:

$$t_{3_\mathrm{e}}(\mathrm{ST})_\mathrm{R} = TT \qquad \text{s} \qquad (13\text{–}92)$$

$$t_{3_\mathrm{s}}(\mathrm{ST})_\mathrm{R} = TT - T_\mathrm{c} \qquad \text{s} \qquad (13\text{–}93)$$

where these epochs, the time tag TT, and the count interval T_c are measured in seconds of station time ST at the receiving electronics (subscript R) at the receiving station on Earth. The epochs (13–92) and (13–93) at the end and start of the count interval T_c for the total-count phase observable could be integer tenths of a second, but in all probability will be integer seconds. Since the accumulated phase $\phi(t_3)$ of the received signal (defined by Eq. 13–34) is measured (in quadruple precision) every tenth of a second, no interpolation of this data is required to evaluate Eq. (13–91). This equation should be calculated in quadruple precision.

13.4.3 COMPUTED VALUES OF TOTAL-COUNT PHASE OBSERVABLES

13.4.3.1 Ramped Two-Way (P_2) and Three-Way (P_3) Total-Count Phase Observables

After the Network Simplification Program (NSP) is implemented, the doppler reference frequency $f_{\mathrm{REF}}(t_3)$ given by Eq. (13–9) will be zero. Hence, computed values of ramped two-way (F_2) or three-way (F_3) doppler observables can be calculated from the second term of Eq. (13–50) or Eq. (13–54). Multiplying this equation by the count interval T_c for total-count phase observables gives the following equation for calculating the computed values of ramped two-way (P_2) or three-way (P_3) total-count phase observables obtained at a tracking station on Earth that has a Block 5 receiver:

$$\text{ramped } P_{2,3} = -\, M_2 \int_{t_{1_s}(\mathrm{ST})_{\mathrm{T}}}^{t_{1_e}(\mathrm{ST})_{\mathrm{T}}} f_{\mathrm{T}}(t_1)\, dt_1 \qquad \text{cycles} \qquad (13\text{–}94)$$

The reception times at the receiving station on Earth at the end and start of the count interval T_c for the total-count phase observable are calculated from Eqs. (13–92) and (13–93). The corresponding epochs at the end and start of the transmission interval T_c' at the transmitting station on Earth, which appear in Eq. (13–94), are calculated from Eqs. (13–53) and (13–52), respectively. These epochs are in station time ST at the transmitting electronics at the transmitting station on Earth. In Eqs. (13–53) and (13–52), ρ_e and ρ_s are the precision round-trip light times (calculated from Eq. 11–7) for the round-trip light-time solutions at the end and start of the count interval for the total-count phase observable. The precision width of the transmission interval in station time ST at the transmitting electronics at the transmitting station on Earth is calculated from Eq. (13–51). For total-count phase observables, this equation is evaluated in quadruple precision.

The integral in Eq. (13–94) can be evaluated using ramp tables as described in Section 13.3.2.2.2 or phase tables as described in Section 13.3.2.2.3. To prevent a loss of precision for the extremely long count intervals that are possible with total-count phase observables, this integral should be evaluated in quadruple precision. Eqs. (13–92) and (13–93) for the end and start of the reception interval T_c are exact in double precision. However, Eq. (13–51) for the precision width of the transmission interval T_c' and Eqs. (13–52) and (13–53) for the start and end of the transmission interval should be evaluated in quadruple precision. If the integral in Eq. (13–94) is evaluated using ramp tables, the algorithm given in Section 13.3.2.2.2 (except Eq. 13–66) should be evaluated in quadruple precision. If the integral in Eq. (13–94) is evaluated using phase tables, the precision used for evaluating the algorithm given in Section 13.3.2.2.3 (which refers to Section 13.2.7) must be changed somewhat from that used in calculating the computed values of doppler observables. For doppler observables, the algorithm is evaluated in double precision, except that differences of interpolated phases in Eqs. (13–13), (13–14), and the last term of Eq. (13–74) are calculated in quadruple precision and then rounded to double precision. For total-count phase observables, Eqs. (13–68) and (13–69) should be evaluated in quadruple precision. The resulting values of Δt_s and Δt_e can then be rounded to double precision. Eq. (13–74), which is the right-hand side of Eq. (13–67) for the integral in Eq. (13–94) should be evaluated in quadruple precision using double precision values of the phase differences $\Delta\phi\,(\Delta t_e)$ and $\Delta\phi\,(\Delta t_s)$. Multiplication of this integral by the spacecraft turnaround ratio M_2 in Eq. (13–94) should be performed in quadruple precision. This gives the computed value of a ramped two-way (P_2) or three-way (P_3) total-count phase observable in quadruple precision.

The media corrections for the computed values of ramped two-way (P_2) and three-way (P_3) total-count phase observables are calculated in the Regres editor from Eq. (13–58) multiplied by the count interval T_c:

$$\Delta\left(\text{ramped } P_{2,3}\right) = M_2 \left[f_T\left(t_{1_e}\right)\Delta\rho_e - f_T\left(t_{1_s}\right)\Delta\rho_s \right] \quad \text{cycles} \qquad (13\text{–}95)$$

The transmitter frequencies $f_T\left(t_{1_s}\right)$ at the start and $f_T\left(t_{1_e}\right)$ at the end of the transmission interval at the transmitting station on Earth are obtained in evaluating the integral in Eq. (13–94) as described above using the algorithm of Section 13.3.2.2.2 or 13.3.2.2.3. The media corrections $\Delta\rho_e$ and $\Delta\rho_s$ to ρ_e and ρ_s, respectively, are calculated in the Regres editor from Eqs. (10–28) and (10–29) as described in Section 10.2. The transmitter frequencies $f_T\left(t_{1_s}\right)$ and $f_T\left(t_{1_e}\right)$ are also used in Eqs. (13–19) and (13–20), which are used in calculating the charged particle contributions to $\Delta\rho_e$ and $\Delta\rho_s$.

The partial derivatives of the computed values of ramped two-way (P_2) and three-way (P_3) total-count phase observables with respect to the solve-for and consider parameter vector $\mathbf{q}$ are calculated from Eq. (13–59) multiplied by the count interval T_c:

$$\frac{\partial\left(\text{ramped } P_{2,3}\right)}{\partial \mathbf{q}} = M_2 \left[f_T\left(t_{1_e}\right) \frac{\partial \rho_e}{\partial \mathbf{q}} - f_T\left(t_{1_s}\right) \frac{\partial \rho_s}{\partial \mathbf{q}} \right] \tag{13–96}$$

The partial derivatives of the precision round-trip light times ρ_e and ρ_s at the end and start of the count interval T_c with respect to the solve-for and consider parameter vector $\mathbf{q}$ are calculated from the formulation given in Section 12.5.1 as described in that section.

In order to calculate the computed value of a total-count phase observable, two light-time solutions are required. The light-time solutions at the end and start of the reception interval T_c at the receiving station on Earth have reception times given by Eqs. (13–92) and (13–93), respectively. Figure 13–2 shows the configuration of count intervals for total-count phase observables during a pass of data at a tracking station on Earth (or for that part of a pass of data for which the accumulated phase of the received signal is continuous). Each new observable requires a new light-time solution at the end of its count interval T_c. However, the light-time solution at the start of the count interval is the same for all data points in the pass (or continuous part of the pass). The common light-time solution at the start of all of the count intervals should be computed for the first data point only. For ramped two-way (P_2) and three-way (P_3) total-count phase observables, the following quantities, computed from this round-trip light-time solution and related calculations should be saved and used in obtaining the computed values, media corrections, and partial derivatives for the remaining data points of the pass (or continuous part of the pass):

$$
\begin{aligned}
&\rho_{\mathrm{s}},\ t_{1_{\mathrm{s}}}(\mathrm{ST})_{\mathrm{T}} \\
&\Delta t_{\mathrm{s}},\ \Delta\phi\left(\Delta t_{\mathrm{s}}\right),\ T_{\mathrm{s}},\ \phi\left(T_{\mathrm{s}}\right) \\
&f_{\mathrm{T}}\left[t_{1_{\mathrm{s}}}(\mathrm{ST})_{\mathrm{T}}\right] \\
&\Delta\rho_{\mathrm{s}} \\
&\frac{\partial\rho_{\mathrm{s}}}{\partial\mathbf{q}}
\end{aligned}
\tag{13–97}
$$

and the auxiliary angles computed on the up and down legs of this light-time solution. The variables $t_{1_{\mathrm{s}}}(\mathrm{ST})_{\mathrm{T}}$, Δt_{s}, and $\phi(T_{\mathrm{s}})$ are quadruple precision. The remaining variables are double precision.

From Eq. (13–91), the standard deviation of the observed value of a one-way (P_1) total-count phase observable or a ramped two-way (P_2) or three-way (P_3) total-count phase observable is given by:

$$
\sigma P_1,\ \sigma\left(\text{ramped } P_{2,3}\right) = \sigma\phi\left(t_{3_{\mathrm{e}}}\right) \qquad \text{cycles} \tag{13–98}
$$

where $\sigma\phi\left(t_{3_{\mathrm{e}}}\right)$ is the standard deviation of the accumulated phase $\phi(t_3)$ of the received signal at the tracking station on Earth. In fitting computed values of total-count phase observables to observed values, true values of $\phi\left(t_{3_{\mathrm{e}}}\right)$ are fit to observed values in a least squares sense.

From Eq. (13–91), the observed values of all total-count phase observables obtained during a pass of data (or that part of the pass for which the accumulated phase of the received signal is continuous) at a tracking station on Earth contain the bias:

$$\Delta P_1,\ \Delta\left(\text{ramped } P_{2,3}\right) = \Delta\phi\left(t_{3_\mathrm{s}}\right) \qquad \text{cycles} \tag{13–99}$$

where $\Delta\phi\left(t_{3_\mathrm{s}}\right)$ is the error in the accumulated phase of the received signal at the common start time for the group of observables. This error can be accounted for by adding it as a solve-for bias parameter to the corresponding computed values of these observables. Then, the partial derivatives of computed values of one-way (P_1) total-count phase observables or ramped two-way (P_2) or three-way (P_3) total-count phase observables with respect to the error in the accumulated phase of the received signal at the common start time for the count intervals are given by:

$$\frac{\partial\left(P_1 \text{ or ramped } P_{2,3}\right)}{\partial\left[\Delta\phi\left(t_{3_\mathrm{s}}\right)\right]} = +1 \tag{13–100}$$

Since an estimate of $\Delta\phi\left(t_{3_\mathrm{s}}\right)$ is obtained for each group of observables having a common start time for their count intervals, the solve-for parameter in the denominator of Eq. (13–100) must contain the group number. The estimated value of the bias $\Delta\phi\left(t_{3_\mathrm{s}}\right)$ will not be added to the computed observable. Hence, when iterating, the estimate of $\Delta\phi\left(t_{3_\mathrm{s}}\right)$ obtained on each iteration will be the total correction.

For each P_2 or P_3 total-count phase observable, the partial derivative (13–100) must be added to the element of the vector (13–96) reserved for $\Delta\phi\left(t_{3_\mathrm{s}}\right)$ for the group of total-count phase observables which contains the P_2 or P_3 data point.

Since the bias $\Delta\phi\left(t_{3_\mathrm{s}}\right)$ in the observed values of total-count phase observables is treated as a solve-for bias parameter, it does not contribute to the standard deviation of these observables given by Eq. (13–98). Hence, the weighting matrix for total-count phase observables is diagonal.

13.4.3.2 One-Way (P_1) Total-Count Phase Observables

After the Network Simplification Program (NSP) is implemented, computed values of one-way (F_1) doppler observables obtained at a tracking station on Earth which has a Block 5 receiver are calculated from Eq. (13–82). Multiplying this equation by the count interval T_c for total-count phase observables gives the following equation for calculating the computed values of one-way (P_1) total-count phase observables obtained at a tracking station on Earth which has a Block 5 receiver:

$$P_1 = -C_2 \left\{ f_{T_0} + \Delta f_{T_0} + f_{T_1}\left(t_{2_m} - t_0\right) + f_{T_2}\left[\left(t_{2_m} - t_0\right)^2 + \frac{1}{12}\left(T_c'\right)^2\right]\right\} T_c' \quad \text{cycles} \qquad (13\text{–}101)$$

The epochs at the end and start of the reception interval T_c at the receiving station on Earth are calculated from Eqs. (13–92) and (13–93). The corresponding transmission times at the spacecraft in coordinate time ET are calculated from Eqs. (13–79) and (13–80). In these equations, ρ_{1_e} and ρ_{1_s} are precision one-way light times calculated from the light-time solutions at the end and start of the count interval T_c. These precision light times are defined by Eq. (11–9) and calculated from Eq. (11–41). The average of the two transmission times at the spacecraft, t_{2_m}, is calculated from Eq. (13–81). The quadratic coefficients Δf_{T_0}, f_{T_1}, and f_{T_2} are assumed to be constant for each group of total-count phase observables. They are selected as the coefficients for the time block containing t_{2_m} for the last data point of the group. The transmission interval T_c' at the spacecraft in seconds of International Atomic Time TAI at the spacecraft is calculated from Eq. (13–77). This equation contains the precision one-way light times $\hat{\rho}_{1_e}$ and $\hat{\rho}_{1_s}$, which are defined by Eq. (11–8). The difference of these light times is calculated from Eq. (11–11) using ρ_{1_e}, ρ_{1_s}, and the parameter Δ, which is defined by Eq. (11–12). The parameter Δ is calculated from Eqs. (11–15) to (11–39) of Section 11.4 using quantities calculated at the transmission times of the light-time solutions at the end and start of the count time T_c.

If the preceding formulation for calculating the parameter Δ were applied independently to each one-way total-count phase observable (P_1) in a pass of data (or the continuous part of a pass) (see Figure 13–2), the calculation of Δ would become increasingly inaccurate as the count interval T_c approached the length of the pass. Hence, the calculation of Δ for each P_1 observable in a pass of data should be modified as follows. For the first data point in a pass of data, the parameter Δ can be computed from the existing formulation. For each data point of the pass, save the parameters I_e, $\dot{I}_e$, and $t_{2_e}(\text{ET})$, which are computed at the end of the count interval T_c for the data point. Then, for each data point of the pass except the first, the values of I_e, $\dot{I}_e$, and $t_{2_e}(\text{ET})$ for the data point and the corresponding values saved from the preceding data point (with each subscript e changed to s) can be substituted into Eqs. (11–17) and (11–18) to give the increment to Δ which has accumulated from the end of the count interval for the preceding data point to the end of the count interval for the current data point. Add this increment for Δ to the value of Δ for the preceding data point to obtain the value of Δ for the current data point.

Eq. (13–77) for the transmission interval $T_c{}'$ at the spacecraft is evaluated in quadruple precision using a double precision value of the change in the precision one-way light time $\hat{\rho}_{1_e} - \hat{\rho}_{1_s}$, which is calculated from Eq. (11–11) and related equations of Section 11.4. Eq. (13–101) is evaluated in quadruple precision using a double precision value of the quadratic offset of the average spacecraft transmitter frequency from its nominal value f_{T_0}.

The media corrections for the computed values of one-way (P_1) total-count phase observables are calculated in the Regres editor from Eq. (13–85) multiplied by the count interval T_c:

$$\Delta P_1 = C_2 f_{S/C}{}^* \left(\Delta\rho_{1_e} - \Delta\rho_{1_s}\right) \qquad \text{cycles} \qquad (13\text{–}102)$$

where $f_{S/C}{}^*$ is given by Eq. (13–86). The media corrections $\Delta\rho_{1_e}$ and $\Delta\rho_{1_s}$ to the precision one-way light times ρ_{1_e} and ρ_{1_s}, respectively, are calculated in the Regres editor from Eqs. (10–24) and (10–25) as described in Section 10.2. The approximate down-leg transmitter frequency given by Eq. (13–21) is used in calculating the charged-particle contributions to $\Delta\rho_{1_e}$ and $\Delta\rho_{1_s}$.

The partial derivatives of the computed values of one-way (P_1) total-count phase observables with respect to the solve-for and consider parameter vector **q** are calculated from Eq. (13–87) multiplied by the count interval T_c:

$$\frac{\partial P_1}{\partial \mathbf{q}} = C_2 f_{S/C}^{*} \left(\frac{\partial \rho_{1_e}}{\partial \mathbf{q}} - \frac{\partial \rho_{1_s}}{\partial \mathbf{q}} \right) \tag{13–103}$$

The partial derivatives of the precision one-way light times ρ_{1_e} and ρ_{1_s} at the end and start of the count interval T_c with respect to the solve-for and consider parameter vector **q** are calculated from the formulation given in Section 12.5.2 as described in that section.

Referring to Figure 13–2, the light-time solution at the start of the count interval is the same for all data points in the pass (or continuous part of the pass). The common light-time solution at the start of all of the count intervals should be computed for the first data point only. For one-way (P_1) total-count phase observables, the following quantities, computed from this one-way light-time solution and related calculations should be saved and used in obtaining the computed values, media corrections, and partial derivatives for the remaining data points of the pass (or continuous part of the pass):

$$\rho_{1_s},\ t_{2_s}(\mathrm{ET})$$

$$C_2 f_{T_0} \tag{13–104}$$

$$\Delta\rho_{1_s},\ \frac{\partial \rho_{1_s}}{\partial \mathbf{q}}$$

and the auxiliary angles computed on this down-leg light-time solution. All of these variables are double precision.

The partial derivatives of the computed values of one-way (P_1) total-count phase observables with respect to the error in the accumulated phase of the received signal at the common start time for the count intervals are given by Eq. (13–100). Since an estimate of $\Delta\phi\left(t_{3_s}\right)$ is obtained for each group of observables having a common start time for their count intervals, the solve-for parameter in the denominator of Eq. (13–100) must contain the group number. For each P_1 total-count phase observable, the partial derivative (13–100) must be added to the element of the vector (13–103) reserved for $\Delta\phi\left(t_{3_s}\right)$ for the group of total-count phase observables which contains the P_1 data point.

The partial derivatives of the computed values of one-way (P_1) total-count phase observables with respect to the quadratic coefficients of the offset of $f_{S/C}$ from f_{T_0} are given by Eqs. (13–88) to (13–90) multiplied by the count interval T_c:

$$\frac{\partial P_1}{\partial \Delta f_{T_0}} = -\,C_2\,T_c{}' \tag{13–105}$$

$$\frac{\partial P_1}{\partial f_{T_1}} = -\,C_2\left(t_{2_m} - t_0\right)T_c{}' \tag{13–106}$$

$$\frac{\partial P_1}{\partial f_{T_2}} = -\,C_2\left[\left(t_{2_m} - t_0\right)^2 + \frac{1}{12}\left(T_c{}'\right)^2\right]T_c{}' \tag{13–107}$$

These partial derivatives must be added to the elements of Eq. (13–103) which are reserved for these parameters.

13.4.4 OBSERVED MINUS COMPUTED RESIDUALS FOR TOTAL-COUNT PHASE OBSERVABLES

Observed values of one-way (P_1) and ramped two-way (P_2) and three-way (P_3) total-count phase observables are calculated from Eq. (13–91) in quadruple precision. Computed values of ramped two-way (P_2) and three-way (P_3) total-count phase observables are calculated from Eq. (13–94) in quadruple precision as described in the second paragraph of Section 13.4.3.1. Computed values of one-way (P_1) total-count phase observables are calculated from Eq. (13–101) in quadruple precision as described in the third paragraph of Section 13.4.3.2.

For one-way (P_1) and ramped two-way (P_2) and three-way (P_3) total-count phase observables, calculate the observed minus computed residuals in quadruple precision. The resulting residuals can then be rounded to double precision and written on the Regres file. After these calculations are completed, the observed and computed values of these observables can be rounded to double precision and written on the Regres file.

13.5 RANGE OBSERVABLES

13.5.1 INTRODUCTION

This section gives the formulation for calculating the observed and computed values of round-trip range observables for three different ranging systems. The Sequential Ranging Assembly (SRA) is the currently operational ranging system. The Planetary Ranging Assembly (PRA) is the previously operational ranging system. The Next-Generation Ranging Assembly (RANG) should be operational by the time the Network Simplification Program (NSP) becomes operational. These observables are measured in range units, which are defined in Section 13.5.2. That section gives the equations for calculating the conversion factor F from seconds to range units at the transmitting and receiving stations.

The observed values of the range observables for the three different ranging systems are defined in Section 13.5.3.1. For each system, the observable can be two-way (same transmitting and receiving stations on Earth) or three-way (different transmitting and receiving stations on Earth). Section 13.5.3.2 gives the equations for calculating the calibrations for these range observables. These calibrations remove small effects contained in the actual observables that are not modelled in the computed observables, which are calculated in program Regres.

Section 13.5.4.1 gives the formulations for calculating the computed values of two-way and three-way SRA, PRA, and RANG range observables. For SRA and PRA, two-way range can be ramped or unramped, and three-way range is ramped. Two-way and three-way RANG range observables are ramped. Calculation of the computed values of these range observables requires the integral of Fdt over an interval of time at the transmitting station and, for three-way SRA or PRA data, the integral of Fdt over an interval of time at the receiving station. The procedure for evaluating these integrals is described in Section 13.5.4.2. The equations for calculating media corrections for these computed observables and partial derivatives of the computed observables with respect to the solve-for and consider parameter vector **q** are given in Section 13.5.4.3.

13.5.2 CONVERSION FACTOR *F* FROM SECONDS TO RANGE UNITS

In order to calculate computed values of range observables, media corrections, partial derivatives, and calibrations for observed values of range observables, the equations for calculating the conversion factor F from seconds to range units at the transmitting and receiving stations are required. The integral of Fdt at the transmitting station gives the change in the phase of the transmitted ranging code (measured in range units) that occurs during an interval of station time ST at the transmitting electronics at the transmitting station on Earth. For three-way SRA or PRA range, the integral of Fdt at the receiving station gives the change in the phase of the transmitter ranging code (measured in range units) which occurs during an interval of station time ST at the receiving electronics at the receiving station on Earth.

The equation for calculating the conversion factor F at the transmitting or receiving station on Earth is a function of the uplink band at the station. For an S-band transmitter frequency $f_T(S)$,

$$F = \frac{1}{2} f_T(S) \qquad \text{range units/second} \qquad (13\text{–}108)$$

Note that one range unit is 2 cycles of the S-band transmitted frequency. For an X-band uplink at a 34-m AZ-EL mount high efficiency (HEF) antenna prior to its conversion to a block 5 exciter (BVE),

$$F = \frac{11}{75} f_T(X, \text{HEF}) \qquad \text{range units/second} \qquad (13\text{–}109)$$

Note that one range unit is $75/11$ cycles of the X-band transmitted frequency at a HEF station prior to its conversion to a block 5 exciter. For an X-band uplink at any tracking station that has a BVE,

$$F = \frac{221}{749 \times 2} f_T(\mathrm{X,BVE}) \qquad \text{range units/second} \qquad (13\text{–}110)$$

Note that one range unit is $(749 \times 2)/221$ cycles of the X-band transmitted frequency at any tracking station that has a BVE.

The ranging formulation given in this section applies for S-band or X-band uplinks at the transmitting and receiving stations on Earth and an S-band or X-band downlink for the data point. The DSN has no current requirements for ranging at other bands (*e.g.*, Ka-band or Ku-band). However, we may be ranging at Ka-band in a few years.

13.5.3 OBSERVED VALUES OF RANGE OBSERVABLES

13.5.3.1 Observed Values

SRA and PRA range observables are obtained from the ranging machine at the receiving station on Earth. These range observables are equal to the phase of the transmitter ranging code at the receiving station minus the phase of the received ranging code. This phase difference is measured in range units at the reception time $t_3(\mathrm{ST})_\mathrm{R}$ in station time ST at the receiving electronics at the receiving station on Earth. The phase difference is measured modulo M range units, where M is the length of the ranging code in range units. It is the period in range units of the lowest frequency ranging component modulated onto the uplink carrier at the transmitting station on Earth. Two-way ranging is obtained using one ranging machine. Three-way ranging requires two ranging machines, one at the transmitting station and one at the receiving station.

Observed and computed values of SRA and PRA range observables are a function of the uplink band at the transmitting station and the uplink band at the receiving station. For three-way data, they can be different. All two-way and three-way PRA range observables were obtained using an S-band uplink at the transmitting station and at the receiving station (the same station for two-way data). Referring to Eqs. (13–108) to (13–110), the X-band exciters at HEF stations (prior to their conversion to Block 5 exciters) are incompatible with Block 3 and

Block 4 S-band exciters and Block 5 S-band and X-band exciters. Hence, Eq. (13–109) can only be used to calculate two-way X-band SRA range obtained from one HEF station (prior to its conversion to a Block 5 exciter) or three-way X-band SRA range obtained from two such stations. On the other hand, three-way SRA range can be obtained using an S-band exciter (Eq. 13–108) or an X-band Block 5 exciter (Eq. 13–110) at the transmitting station and at the receiving station. All four band combinations are possible (*i.e.*, S-band uplink bands at both stations, X-band uplink bands at both stations, an S-band uplink at the transmitting station and an X-band uplink at the receiving station, and an X-band uplink at the transmitting station and an S-band uplink at the receiving station).

The ranging code is modulated onto the uplink carrier at the transmitting station on Earth. The spacecraft multiplies the frequency of the received signal by the spacecraft transponder turnaround ratio M_2 and then remodulates the ranging code onto the downlink carrier. The uplink and downlink carriers and range codes are phase coherent. Hence, it will be seen in Section 13.5.4 that the spacecraft transponder turnaround ratio M_2 is not used in calculating the computed values of range observables. However, in calculating media corrections for computed range observables, the down-leg charged-particle correction requires the transmitter frequency for the down leg which is calculated from Eq. (13–20). This equation does contain the spacecraft turnaround ratio M_2. This is the only place where M_2 is used in processing round-trip range observables.

The Next-Generation Ranging Assembly (RANG) measures the phase of the transmitted ranging code at the transmitting station on Earth and the phase of the received ranging code at the receiving station on Earth. The phases of the transmitted and received ranging codes are measured independently in range units (modulo M range units) approximately every ten seconds. The time tags for the transmitted phases are seconds of station time ST at the transmitting electronics at the transmitting station on Earth. The time tags for the received phases are seconds of station time ST at the receiving electronics at the receiving station on Earth. The transmitter signal at the receiving station on Earth is not used for this data type. To be consistent with the definition of SRA and PRA range observables, RANG range observables are defined to be the negative of the phase of the received ranging code at the receiving station on Earth. The phase of the transmitted ranging code at the transmitting station on Earth is used in program Regres to calculate the computed values of these observables.

RANG range observables are a function of the uplink band at the transmitting station.

13.5.3.2 Calibrations

The information content in range observables is in the phase of the received ranging code at the receiving electronics at the receiving station on Earth. The phase of the received ranging code is the same as the phase of the transmitted ranging code at the transmitting electronics at the transmitting station on Earth one round-trip light time earlier. The actual round-trip light time contains delays in the transmitting and receiving electronics and in the spacecraft transponder. These delays affect the range observables. However, they are not modelled in program Regres and hence their effects are not included in the computed values of the range observables. Hence, in this section we develop equations for corrections to range observables which remove the effects of the unmodelled delays from the range observables.

The delays at the transmitter, spacecraft, and receiver change the transmission time at the transmitting electronics at the transmitting station on Earth by $\Delta t_1(\mathrm{ST})_\mathrm{T}$ seconds. This changes the phase of the transmitted signal at the transmitting electronics by $F\left[t_1(\mathrm{ST})_\mathrm{T}\right]\Delta t_1(\mathrm{ST})_\mathrm{T}$ range units, where the conversion factor F from seconds to range units is given by Eq. (13–108), (13–109), or (13–110). The equation selected depends upon the uplink band at the transmitting station and the type of the exciter. The change in the transmission time is the negative of the change in the round-trip light time $\Delta\rho$, which is the sum of the delays. Hence, the change in the phase of the transmitted signal is $-F\left[t_1(\mathrm{ST})_\mathrm{T}\right]\Delta\rho$. The change in the phase of the received signal at the receiving electronics at the receiving station on Earth is the same. But, all round-trip range observables contain the negative of the phase of the received signal. Hence, the effect of the delays at the transmitter, spacecraft, and receiver on the observed values ρ(RU) of SRA, PRA, and RANG range observables in range units is given by:

$$\Delta\rho(\mathrm{RU}) = F\left[t_1(\mathrm{ST})_\mathrm{T}\right]\Delta\rho \qquad \text{range units} \qquad (13\text{–}111)$$

This effect must be subtracted from the observed values of all SRA, PRA, and RANG range observables.

The sum $\Delta\rho$ of the delays at the transmitting station, spacecraft, and receiving station in seconds is calculated from:

$$\begin{aligned}\Delta\rho = {} & Cal_{\mathrm{RCVR}} / 2 - Zcorr_{\mathrm{RCVR}} / 2 \\ & + S / C_{delay} \qquad\qquad \mathrm{s} \qquad (13\text{–}112) \\ & + Cal_{\mathrm{XMTR}} / 2 - Zcorr_{\mathrm{XMTR}} / 2\end{aligned}$$

The term Cal_{RCVR} is the measured round-trip delay at the receiving station on Earth from the receiving electronics to the Test Translator. The term $Zcorr_{\mathrm{RCVR}}$ is the round-trip delay to the Test Translator minus the round-trip delay from the receiving electronics to the tracking point. Hence, Cal_{RCVR} minus $Zcorr_{\mathrm{RCVR}}$ is the round-trip delay from the receiving electronics to the tracking point at the receiving station on Earth. It is divided by two to approximate the down-leg delay at the receiver. Line three of Eq. (13–112) contains the corresponding terms, which approximate the up-leg delay at the transmitting station on Earth. The term on the second line of Eq. (13–112) is the delay in the spacecraft transponder. The tracking points of the transmitting and receiving antennas are the secondary axes of these antennas.

For a tracking station that has its electronics located close to the antenna, the measured round-trip delay Cal_{RCVR} or Cal_{XMTR} is used directly in Eq. (13–112). However, some stations have the transmitting and receiving electronics located tens of kilometers away from the antenna. If the receiving station has remote electronics, the nominal value τ_{D} of the downlink delay is passed to Regres on the OD file and $Cal_{\mathrm{RCVR}} / 2$ in Eq. (13–112) is replaced by $Cal_{\mathrm{RCVR}} / 2$ minus τ_{D}. Similarly, if the transmitting station has remote electronics, the nominal value τ_{U} of the uplink delay is passed to Regres on the OD file, and $Cal_{\mathrm{XMTR}} / 2$ in Eq. (13–112) is replaced by $Cal_{\mathrm{XMTR}} / 2$ minus τ_{U}. Program Regres uses the nominal value τ_{D} of the downlink delay and the nominal value τ_{U} of the uplink delay to perform the round-trip spacecraft light-time solution and calculate the precision round-trip light time from Eq. (11–7).

If the received signal at a tracking station on Earth is a carrier-arrayed signal obtained by combining signals from several antennas, it will contain a fixed delay on the order of 1 ms. This delay does not affect the calculation of the range calibration from Eqs. (13–111) and (13–112) as described above. However, the downlink delay passed to program Regres on the OD file is the nominal value τ_{D} described above plus the carrier-arrayed delay (see Section 11.2).

Eq. (13–111) is evaluated in the ODE using the ODE's approximation for the round-trip light time. Evaluation of the transmitter frequency in Eq. (13–108), (13–109), or (13–110) is accomplished by interpolating the ramp table or the phase table for the transmitting station on Earth as described in Sections 13.2.6 and 13.2.7. If a delay in Eq. (13–112) is available in range units instead of seconds, then that term should not be multiplied by $F\left[t_1(\mathrm{ST})_{\mathrm{T}}\right]$ in Eq. (13–111).

After subtracting the range calibration given by Eq. (13–111) from the observed values of SRA, PRA, and RANG range observables, the resulting observed values of SRA and PRA range observables should be greater than or equal to zero and less than M range units. The resulting observed values of RANG range observables should be greater than $-M$ range units and less than or equal to zero. Add or subtract as necessary M range units until the corrected observables are within these ranges. The equations for calculating the length M of the ranging code in range units are given in Section 13.5.4.1.

13.5.4 COMPUTED VALUES OF RANGE OBSERVABLES, MEDIA CORRECTIONS, AND PARTIAL DERIVATIVES

13.5.4.1 Computed Values of Range Observables

The equations for calculating the computed values of three-way ramped, two-way ramped, and two-way unramped SRA and PRA range observables follow from the definition of the observed values of these data types given in Section 13.5.3.1. The computed values of three-way ramped SRA and PRA range observables are calculated from:

$$\rho_3\left(\text{ramped}\right) = \left[\int_{T_B}^{t_3(\text{ST})_R} F\left(t_3\right) dt_3 - \int_{T_A}^{t_1(\text{ST})_T} F\left(t_1\right) dt_1\right], \text{ modulo } M \qquad \text{range units} \qquad (13\text{–}113)$$

The reception time $t_3(\text{ST})_R$ in station time ST at the receiving electronics at the receiving station on Earth is equal to the data time tag TT:

$$t_3(\text{ST})_R = TT \qquad \text{s} \qquad (13\text{–}114)$$

The corresponding transmission time $t_1(\text{ST})_T$ in station time ST at the transmitting electronics at the transmitting station on Earth is calculated from:

$$t_1(\text{ST})_T = t_3(\text{ST})_R - \rho \qquad \text{s} \qquad (13\text{–}115)$$

where ρ is the precision round-trip light time defined by Eq. (11–5). It is calculated from the round-trip light-time solution using Eq. (11–7). The quantities T_B and T_A are zero-phase times at the receiving and transmitting stations, respectively. At T_B, the phase of the transmitter ranging code at the receiving station is zero. The phase of the transmitted ranging code at the transmitting station is zero at T_A. The conversion factor $F(t_3)$ at the receiving station and $F(t_1)$ at the transmitting station are calculated from Eq. (13–108), (13–109), or (13–110), depending upon the uplink band and exciter type at the station. The precision width W of the interval of integration at the receiving station is given by:

$$W = t_3(\mathrm{ST})_\mathrm{R} - T_\mathrm{B} \qquad \text{s} \qquad (13\text{–}116)$$

The precision width W of the interval of integration at the transmitting station is given by:

$$W = \left[t_3(\mathrm{ST})_\mathrm{R} - T_\mathrm{A}\right] - \rho \qquad \text{s} \qquad (13\text{–}117)$$

The first integral in Eq. (13–113) is the phase of the transmitter ranging code at the reception time $t_3(\mathrm{ST})_\mathrm{R}$ at the receiving electronics at the receiving station on Earth. The second integral in Eq. (13–113) is the phase of the transmitted ranging code at the transmission time $t_1(\mathrm{ST})_\mathrm{T}$ at the transmitting electronics at the transmitting station on Earth. It is equal to the phase of the received ranging code at $t_3(\mathrm{ST})_\mathrm{R}$. The phases of the transmitter ranging code and the received ranging code at the reception time $t_3(\mathrm{ST})_\mathrm{R}$ at the receiving electronics at the receiving station on Earth are measured in range units. The difference of these two phases is calculated modulo M range units, where M is the length of the ranging code in range units. For SRA range, the modulo number M is calculated from:

$$M = 2^{n+6} \qquad \text{range units} \qquad (13\text{–}118)$$

where n is the component number of the lowest frequency ranging component, which is the highest component number. For PRA range, M is calculated from:

$$M = 2^{9 + \mathrm{minimum}(1, HICOMP) + LOWCOMP} \qquad \text{range units} \qquad (13\text{–}119)$$

The component number n for SRA range data and *HICOMP* and *LOCOMP* for PRA range data are non-negative integers obtained from the record of the OD file for the data point.

Eq. (13–113) for the computed values of three-way ramped SRA and PRA range observables also applies for two-way ramped SRA and PRA range observables. However, for this case, there is only one tracking station and $T_B = T_A$. Hence, for calculating the computed values of two-way ramped SRA and PRA range observables, Eq. (13–113) reduces to:

$$\rho_2\left(\text{ramped}\right) = \int_{t_1(\text{ST})_\text{T}}^{t_3(\text{ST})_\text{R}} F(t)\, dt\,, \quad \text{modulo } M \qquad \text{range units} \tag{13–120}$$

The reception time $t_3(\text{ST})_\text{R}$ in station time ST at the receiving electronics at the tracking station on Earth and the transmission time $t_1(\text{ST})_\text{T}$ in station time ST at the transmitting electronics at the same tracking station are calculated from Eqs. (13–114) and (13–115). The conversion factor $F(t)$ at the tracking station is calculated from Eq. (13–108), (13–109), or (13–110), depending upon the uplink band and exciter type at the tracking station. The precision width W of the interval of integration at the tracking station is given by:

$$W = \rho \qquad \text{s} \tag{13–121}$$

Eq. (13–120) for the computed values of two-way ramped SRA and PRA range observables also applies for two-way unramped SRA and PRA range observables. If the transmitter frequency is constant during the round-trip light time, the conversion factor $F(t)$ will have a constant value F, and Eq. (13–120) reduces to:

$$\rho_2\left(\text{unramped}\right) = F \times \rho\,, \quad \text{modulo } M \qquad \text{range units} \tag{13–122}$$

The conversion factor F is calculated from Eq. (13–108), (13–109), or (13–110), depending upon the uplink band and exciter type at the tracking station. In these equations, the constant value of the transmitter frequency f_T at the tracking station is obtained from the record of the OD file for the data point (see Section 13.2.1).

From Section 13.5.3.1, the observed values of Next-Generation Ranging Assembly (RANG) range observables are equal to the negative of the phase of the received range code at the receiving electronics at the receiving station on Earth, measured in range units, modulo M range units. The computed values of RANG range observables are equal to the negative of the phase of the transmitted range code at the transmitting electronics at the transmitting station on Earth. The observed received phase and the calculated transmitted phase should be equal. The data time tag TT is the reception time $t_3(\mathrm{ST})_\mathrm{R}$ at the receiving electronics (Eq. 13–114). Given this reception time, the round-trip spacecraft light-time solution is performed, and the precision round-trip light time ρ defined by Eq. (11–5) is calculated from Eq. (11–7). Given $t_3(\mathrm{ST})_\mathrm{R}$ and ρ, the transmission time $t_1(\mathrm{ST})_\mathrm{T}$ at the transmitting electronics at the transmitting station on Earth is calculated from Eq. (13–115).

The OD file will contain range phase records, which will contain range phase tables. Each range phase table contains a sequence of (range phase)-time points. Each point gives the double-precision phase of the transmitted range code in range units (modulo M range units) and the corresponding value of the transmission time in station time ST at the transmitting electronics at a particular tracking station on Earth. Given the transmission time $t_1(\mathrm{ST})_\mathrm{T}$ for a RANG range observable, program Regres will read the range phase table for the transmitting station and select the phase-time point whose transmission time T_E is closest to $t_1(\mathrm{ST})_\mathrm{T}$. The phase of the transmitted range code at T_E is denoted as $\psi_\mathrm{E}(T_\mathrm{E})$.

Given the above quantities, the computed value of a two-way or three-way ramped RANG range observable is calculated from:

$$\rho_{2,3}(\text{ramped}) = -\left\{\left[\psi_\mathrm{E}(T_\mathrm{E}) + \int_{T_\mathrm{E}}^{t_1(\mathrm{ST})_\mathrm{T}} F(t_1)\,dt_1\right],\ \text{modulo}\ M\right\}$$

range units (13–123)

The conversion factor $F(t_1)$ at the transmitting station is calculated from Eq. (13–108), (13–109), or (13–110), depending upon the uplink band and exciter type at the transmitting station. For RANG range observables, the modulo number M is calculated from Eq. (13–118) if the ranging code is generated sequentially (*i.e.*, sequential ranging). However, if a pseudo noise (PN) ranging code is used (*i.e.*, pseudo noise ranging), the modulo number M (which will be an integer) will be obtained from the data record for the data point on the OD file. The precision width W of the interval of integration is calculated from:

$$W = \left[t_3(\mathrm{ST})_\mathrm{R} - T_\mathrm{E} \right] - \rho \qquad \mathrm{s} \qquad (13\text{–}124)$$

The integral in Eq. (13–123) is evaluated using the ramp table or the (carrier) phase table for the transmitting station as described in Section 13.5.4.2.

13.5.4.2 Evaluation of Integrals

The two integrals in Eq. (13–113), the integral in Eq. (13–120), and the integral in Eq. (13–123) can be evaluated using ramp tables as described in Section 13.3.2.2.2 or phase tables as described in Section 13.3.2.2.3. In each of the four integrals, the lower limit and the upper limit of the interval of integration are denoted as t_s and t_e, respectively, in the ramp table algorithm. In the phase table algorithm, they are denoted as $t_{1_\mathrm{s}}(\mathrm{ST})_\mathrm{T}$ and $t_{1_\mathrm{e}}(\mathrm{ST})_\mathrm{T}$, respectively. Each algorithm requires the precision width W of the interval of integration. For the four integrals listed above, the corresponding precision widths W are given by Eqs. (13–116), (13–117), (13–121), and (13–124), respectively. In the phase table algorithm, the precision width W is denoted as T_c'.

The algorithms in Sections 13.3.2.2.2 and 13.3.2.2.3 give the time integral of the ramped transmitter frequency f_T, whereas we want the time integral of the conversion factor F. Hence, after evaluating the integral of $f_\mathrm{T}\,dt$, the resulting integral must be multiplied by $1/2$ if F is given by Eq. (13–108), $11/75$ if F is given by Eq. (13–109), and $221/(749 \times 2)$ if F is given by Eq. (13–110).

13.5.4.3 Media Corrections and Partial Derivatives

From Eq. (13–113) for three-way ramped SRA and PRA range observables, Eq. (13–120) for two-way ramped SRA and PRA range observables, and Eq. (13–123) for two-way and three-way ramped RANG range observables, the change $\Delta\rho$(RU) in the computed value of the range observable due to the change $\Delta t_1(\mathrm{ST})_\mathrm{T}$ in the transmission time at the transmitting electronics due to media corrections is given by:

$$\Delta\rho(\mathrm{RU}) = -\; F\left[t_1(\mathrm{ST})_\mathrm{T}\right]\Delta t_1(\mathrm{ST})_\mathrm{T} \qquad \text{range units} \qquad (13\text{–}125)$$

The round-trip light time ρ is defined by Eq. (11–5). Hence, the media correction $\Delta\rho$ to the round-trip light time is the negative of the change in the transmission time due to media corrections:

$$\Delta\rho = -\;\Delta t_1(\mathrm{ST})_\mathrm{T} \qquad \mathrm{s} \qquad (13\text{–}126)$$

Substituting Eq. (13–126) into Eq. (13–125) gives:

$$\Delta\rho(\mathrm{RU}) = F\left[t_1(\mathrm{ST})_\mathrm{T}\right]\Delta\rho \qquad \text{range units} \qquad (13\text{–}127)$$

From Eq. (13–122), Eq. (13–127) also applies for two-way unramped SRA and PRA range observables. However, for this case, the conversion factor F is constant. Hence, Eq. (13–127) gives media corrections for computed values of ramped and unramped two-way and three-way SRA and PRA range observables and ramped two-way and three-way RANG range observables. The media correction $\Delta\rho$ to the round-trip light time ρ is calculated from Eq. (10–27) as described in Section 10.2.

Evaluation of the integrals in Eqs. (13–113), (13–120), and (13–123) using the algorithm in Section 13.3.2.2.2 or Section 13.3.2.2.3 gives the transmitted frequency $f_\mathrm{T}\left[t_1(\mathrm{ST})_\mathrm{T}\right]$ at the transmission time $t_1(\mathrm{ST})_\mathrm{T}$ in station time ST at the transmitting electronics at the transmitting station on Earth. For unramped two-way range (Eq. 13–122), the constant value of the transmitted frequency is obtained from the record of the OD file for the data point. The transmitted frequency $f_{\mathrm{T}}\left[t_1(\mathrm{ST})_{\mathrm{T}}\right]$ is used to calculate the conversion factor $F\left[t_1(\mathrm{ST})_{\mathrm{T}}\right]$ in Eq. (13–127) from Eq. (13–108), (13–109), or (13–110). The transmitted frequency $f_{\mathrm{T}}\left[t_1(\mathrm{ST})_{\mathrm{T}}\right]$ is also used to calculate the up-leg and down-leg transmitted frequencies from Eqs. (13–19) and (13–20). These frequencies are used in calculating the charged-particle contributions to the media correction $\Delta\rho$.

By replacing corrections with partial derivatives in the first paragraph of this section, partial derivatives of computed values of two-way and three-way ramped and unramped SRA, PRA, and RANG range observables with respect to the solve-for and consider parameter vector $\mathbf{q}$ are given by:

$$\frac{\partial\rho(\mathrm{RU})}{\partial\mathbf{q}} = F\left[t_1(\mathrm{ST})_{\mathrm{T}}\right]\frac{\partial\rho}{\partial\mathbf{q}} \qquad (13\text{–}128)$$

The partial derivatives of the round-trip light time ρ with respect to the parameter vector $\mathbf{q}$ are calculated from the formulation of Section 12.5.1.

13.6 GPS/TOPEX PSEUDO-RANGE AND CARRIER-PHASE OBSERVABLES

This section gives the formulation for calculating the observed and computed values of GPS/TOPEX pseudo-range and carrier-phase observables. These are one-way data types. The transmitter is a GPS Earth satellite (semi-major axis $a \approx 26{,}560$ km), and the receiver is either the TOPEX (or equivalent) Earth satellite ($a \approx 7712$ km) or a GPS receiving station on Earth.

Pseudo-range observables are one-way range observables, measured in kilometers. They are equal to the one-way light time multiplied by the speed of light c. Carrier-phase observables are a precise measure of the one-way range in kilometers (light time multiplied by c) plus an unknown bias. The formulation for the computed values of pseudo-range and carrier-phase observables is the same. It contains a bias parameter, which is estimated independently for these two data types. In general, the pseudo-range bias is a small number, and the carrier-phase bias is a large number. Fitting to pseudo-range and carrier-phase observables gives a precise measure of the one-way range throughout a pass of tracking data.

Section 13.6.1 defines the observed values of pseudo-range and carrier-phase observables. The formulation for the computed values of these observables is specified (mainly by reference to Section 11.5) in Section 13.6.2.1. Section 13.6.2.2 gives the formulation for calculating media corrections and partial derivatives for the computed values of these observables.

13.6.1 OBSERVED VALUES

The observed values of GPS/TOPEX pseudo-range and carrier-phase observables are defined in Section 3 of Sovers and Border (1990).

The transmitting GPS satellite modulates a pseudo-random noise ranging code onto the transmitted carrier. A local copy of this ranging code is generated at the receiver. Correlation of the received ranging code with the local copy of the ranging code gives the phase difference of the two ranging codes in cycles of the ranging code. This phase difference is converted to seconds and multiplied by the speed of light c to give the observed pseudo range in kilometers. The mathematical definition of this observable is given by Eq. (11–42) or (11–43).

The carrier frequency transmitted at the GPS satellite is constant. A reference signal with this same constant frequency is generated at the receiver. From Eq. (3.13) of Sovers and Border (1990), the observed value of the carrier-phase observable is the measured phase of the reference signal minus the measured phase of the received signal. This phase difference in cycles of the carrier frequency is then divided by the carrier frequency and multiplied by the speed of light c to give the carrier-phase observable in kilometers. The mathematical definition of this observable is given by Eq. (11–42) or (11–43).

In Eq. (11–42) and (11–43), the light time from the GPS satellite to the receiver (the TOPEX satellite or a GPS receiving station on Earth) should be supplemented with the estimable range bias (in seconds) discussed above. The estimated value of this bias will be a large negative number for carrier-phase observables because the value of the first carrier-phase observable at the start of a pass of data is determined modulo one cycle of the carrier phase. That is, carrier-phase observables, which are continuous throughout each pass of data, start with a value of approximately zero at the start of each pass of data, instead of the actual range at the start of the pass. The time tag for each pseudo-range and carrier-phase observable is the reception time $t_3(\mathrm{ST})_\mathrm{R}$ in station time ST at the receiving electronics at the TOPEX satellite or the GPS receiving station on Earth (Eq. 13–114).

The pseudo-range and carrier-phase observables come in pairs. Each pair consists of one observable obtained from the L1-band transmitter frequency and a second observable obtained from the L2-band transmitter frequency. The two observables of each pair have the same time tag. Each observable pair is used to construct a weighted average observable which is free of the effects of charged particles. What this means is that when the media correction for the computed value of a pseudo-range or carrier-phase observable is calculated, the down-leg charged-particle correction will be zero. The L1-band and L2-band transmitter frequencies are given by Eq. (7–1). The weighting equations are Eqs. (7–2) to (7–4).

13.6.2 COMPUTED VALUES, MEDIA CORRECTIONS, AND PARTIAL DERIVATIVES

13.6.2.1 Computed Values

The first step in calculating the computed value of a GPS/TOPEX pseudo-range or carrier-phase observable is to obtain the down-leg spacecraft light-time solution with the reception time $t_3(\mathrm{ST})_\mathrm{R}$ (the time tag for the data point) in station time ST at the receiving electronics at the TOPEX satellite or the GPS receiving station on Earth. The algorithm for the spacecraft light-time solution is given in Section 8.3.6. The spacecraft light-time solution can be performed in the Solar-System barycentric space-time frame of reference or in the local geocentric space-time frame of reference. This latter frame of reference was added to the ODP specifically for processing GPS/TOPEX data. It can be used when all of the participants are very near to the Earth.

The definitions of GPS/TOPEX pseudo-range and carrier-phase observables are given in Section 13.6.1. From these definitions, the mathematical definition for either of these observables is given by Eq. (11–42) or (11–43). However, as discussed in Section 13.6.1, the down-leg light time in these equations must be supplemented with the estimable range bias (in seconds) discussed in that section. Given the down-leg spacecraft light-time solution, the computed value ρ_1 of a GPS/TOPEX pseudo-range or carrier-phase observable in kilometers, which is defined by Eq. (11–42) or (11–43), is calculated from Eq. (11–44) and related equations as described in Section 11.5.

The estimable bias *Bias* in Eq. (11–44) is estimated independently for pseudo-range and carrier-phase observables as described in Section 11.5.2.

The observed values of pseudo-range and carrier-phase observables are computed as a weighted average, which eliminates the effects of charged particles on the down-leg light time. However, the computed values of pseudo-range and carrier-phase observables still contain three frequency-dependent terms. These terms are computed as a weighted average of the L1-band and L2-band values of these terms using Eqs. (7–2) to (7–4).

The constant phase-center offsets at the GPS receiving station on Earth and at the receiving TOPEX satellite are calculated in Step 2 of the spacecraft light-time solution (Section 8.3.6) using the algorithms given in Sections 7.3.1 and 7.3.3, respectively. The constant phase-center offset at the transmitting GPS satellite is calculated in Step 9 of the spacecraft light-time solution.

Eq. (11–44) for the computed value ρ_1 of a GPS/TOPEX pseudo-range or carrier-phase observable contains a variable phase-center offset $\Delta_{\mathrm{A}}\rho(t_3)$ at the receiver (the TOPEX satellite or a GPS receiving station on Earth) and $\Delta_{\mathrm{A}}\rho(t_2)$ at the transmitting GPS satellite. These variable phase-center offsets are calculated for carrier-phase observables only using the algorithm given in Section 11.5.4.

Eq. (11–44) for ρ_1 also contains a geometrical phase correction $\Delta\Phi$, which is described in Section 11.5.2. It is calculated for carrier-phase observables only using the algorithm given in Section 11.5.3.

The remaining terms of Eq. (11–44) are not frequency dependent and hence do not need to be computed as a weighted average. That is, they are only computed once.

13.6.2.2 Media Corrections and Partial Derivatives

The media correction $\Delta\rho_1$(km) in kilometers to the computed value ρ_1 of a GPS/TOPEX pseudo-range or carrier-phase observable in kilometers is calculated in the Regres editor from:

$$\Delta\rho_1(\mathrm{km}) = \Delta\rho_1(\mathrm{s})\times c \qquad \mathrm{km} \qquad (13\text{–}129)$$

where c is the speed of light and $\Delta\rho_1$(s) is the media correction to the down-leg light time calculated from Eq. (10–26) as described in the last paragraph of Section 10.2.3.2.1 and in Section 10.2.

The partial derivative of the computed value ρ_1 of a GPS/TOPEX pseudo-range or carrier-phase observable with respect to the solve-for and consider parameter vector $\mathbf{q}$ is calculated as described in Section 12.5.3.

13.7 SPACECRAFT INTERFEROMETRY OBSERVABLES

Subsection 13.7.1 gives the formulation for the observed and computed values of narrowband spacecraft interferometry (*INS*) observables, media corrections for the computed observables, and partial derivatives of the computed values of the observables with respect to the solve-for and consider parameter vector **q**. It will be seen that a narrowband spacecraft interferometry observable is equivalent to the difference of two doppler observables received simultaneously at two different tracking stations on Earth.

Subsection 13.7.2 gives the formulation for the observed and computed values of wideband spacecraft interferometry (*IWS*) observables, media corrections for the computed observables, and partial derivatives of the computed values of the observables with respect to the solve-for and consider parameter vector **q**. It will be seen that a wideband spacecraft interferometry observable is equivalent to the difference of two range observables (actually the corresponding light times) received simultaneously at two different tracking stations on Earth.

In the above two paragraphs, the differenced doppler and range observables are actually received at the same value of station time ST at the receiving electronics at two different tracking stations on Earth. Since the ST clocks at the two different tracking stations are not exactly synchronized, the differenced doppler and differenced range observables are not exactly simultaneous.

A deep-space probe can be navigated by using $\Delta VLBI$, which is a narrowband or wideband spacecraft interferometry observable minus a narrowband or wideband quasar interferometry observable, and other data types. This section gives the formulation for spacecraft interferometry observables and Section 13.8 gives the formulation for quasar interferometry observables. The data records for the spacecraft and the quasar interferometry data points are placed onto the Regres file. The differencing of these data types to form $\Delta VLBI$ observables is done in a Regres post processor. Spacecraft and quasar interferometry observables and their difference, $\Delta VLBI$ observables, are only processed in the Solar-System barycentric space-time frame of reference.

13.7.1 NARROWBAND SPACECRAFT INTERFEROMETRY (*INS*) OBSERVABLES

Section 13.7.1.1 describes the actual observed quantities and shows how these quantities are assembled to form the observed values of narrowband spacecraft interferometry observables. It is also shown that a narrowband spacecraft interferometry observable is equivalent to the difference of two doppler observables received at the same value of station time ST at the receiving electronics at two different tracking stations on Earth. If the spacecraft is the transmitter, the doppler observables are one-way. If a tracking station on Earth is the transmitter, the doppler observables are round-trip (*i.e.*, two-way or three-way doppler).

The formulation for the computed values of narrowband spacecraft interferometry observables is given in Section 13.7.1.2.1. The formulations for the media corrections for the computed values of these observables and the partial derivatives of the computed values of these observables with respect to the solve-for and consider parameter vector **q** are given in Sections 13.7.1.2.2 and 13.7.1.2.3, respectively.

13.7.1.1 Observed Values of Narrowband Spacecraft Interferometry (*INS*) Observables

Correlation of a spacecraft signal received on a VLBI (Very Long Baseline Interferometry) receiver with a local model gives the continuous phase of the received signal plus several additional terms, which are functions of station time ST or are constant. When these augmented phases obtained at two different tracking stations on Earth at the same value of station time ST (the ST clocks at the two tracking stations may be synchronized to about the microsecond level) are subtracted, the additional terms cancel. The resulting "observed quantity" is the phase of the received signal at one tracking station at a given value of station time ST minus the phase of the received signal at a second tracking station at the same value (clock reading) of station time ST at that station. This observed phase difference will be in error by an integer number of cycles, which will be constant during a pass of data.

Let

$\left(\phi_2 - \phi_1\right)$ = phase of received spacecraft carrier signal at receiving electronics at tracking station 2 on Earth at station time ST at station 2 minus phase of received spacecraft carrier signal at receiving electronics at tracking station 1 on Earth at the same value of ST at station 1 (cycles). This phase difference is continuous over a pass of data and is in error by a constant integer number of cycles.

A narrowband spacecraft interferometry (*INS*) observable is calculated from the following two observed phase differences:

$\left(\phi_2 - \phi_1\right)_{\mathrm{e}}$ = observed value of $\left(\phi_2 - \phi_1\right)$ at station time ST equal to the data time tag *TT* plus one-half of the count interval T_{c}.

$\left(\phi_2 - \phi_1\right)_{\mathrm{s}}$ = observed value of $\left(\phi_2 - \phi_1\right)$ at station time ST equal to the data time tag TT minus one-half of the count interval T_{c}.

Given the reception times in station time ST for $\left(\phi_2 - \phi_1\right)_{\mathrm{e}}$ and $\left(\phi_2 - \phi_1\right)_{\mathrm{s}}$, the time tag TT for the corresponding *INS* observable is the average of these reception times, and the count interval T_{c} for the *INS* observable is the difference of these reception times. For a pass of *INS* data, the configuration of the count intervals can be in the doppler mode as shown in Figure 13–1 or in the phase mode as shown in Figure 13–2.

A narrowband spacecraft interferometry (*INS*) observable is calculated from the phase difference $\left(\phi_2 - \phi_1\right)_{\mathrm{e}}$ at the end of the count interval minus the phase difference $\left(\phi_2 - \phi_1\right)_{\mathrm{s}}$ at the start of the count interval. In order to be equivalent to differenced doppler, we must divide the change in this phase difference by the count interval T_c. Also, it will be seen that we must change the sign of the resulting quantity. The observed value of a narrowband spacecraft interferometry (*INS*) observable is calculated in the ODE from:

$$INS = -\frac{1}{T_c}\left[\left(\phi_2 - \phi_1\right)_e - \left(\phi_2 - \phi_1\right)_s\right] \qquad \text{Hz} \qquad (13\text{–}130)$$

The following discussion will show that this equation is equivalent to a doppler observable received at tracking station 2 on Earth minus the corresponding doppler observable received at tracking station 1 on Earth. The time tag TT and count interval T_c for each of these doppler observables is the same as the time tag and count interval for the *INS* observable. Let $F1_2$, $F2_2$, and $F3_2$ denote one-way, two-way, and three-way doppler observables received at tracking station 2 on Earth. Also, let $F1_1$, $F2_1$, and $F3_1$ denote one-way, two-way, and three-way doppler observables received at tracking station 1 on Earth. Using these variables, the proposed definition (which remains to be proven correct) of an *INS* observable calculated in the ODE is given by:

$$\begin{aligned} INS &= F1_2 - F1_1 \quad \text{if the spacecraft is the transmitter} \\ &= F2_2 - F3_1 \quad \text{if station 2 is the transmitter} \\ &= F3_2 - F2_1 \quad \text{if station 1 is the transmitter} \\ &= F3_2 - F3_1 \quad \text{if a third station is the transmitter} \end{aligned} \qquad \text{Hz} \qquad (13\text{–}131)$$

If the transmitter is a tracking station on Earth and the transmitter frequency $f_T(t_1)$ is constant, two-way doppler (F_2) and three-way doppler (F_3) in Eq. (13–131) are unramped doppler, which is defined by Eq. (13–31). If Eq. (13–31) is substituted into Eq. (13–131), the first term of Eq. (13–31), which is the same for each of the two round-trip unramped doppler observables, cancels in Eq. (13–131), and the second term of Eq. (13–31) produces Eq. (13–130).

If the transmitter is a tracking station on Earth and the transmitter frequency $f_T(t_1)$ is ramped, two-way doppler (F_2) and three-way doppler (F_3) in Eq. (13–131) are ramped doppler. After the Network Simplification Program is implemented, the definition of one-way (F_1) doppler and ramped two-way (F_2) and three-way (F_3) doppler observables is given by Eq. (13–41). Substituting this equation into Eq. (13–131) gives Eq. (13–130).

Hence, the proposed definition (13–131) of narrowband spacecraft interferometry (*INS*) observables calculated in the ODE from Eq. (13–130) is correct if unramped two-way (F_2) and three-way (F_3) doppler observables in Eq. (13–131) are defined by Eq. (13–31), and one-way (F_1) doppler observables and ramped two-way (F_2) and three-way (F_3) doppler observables in Eq. (13–131) are defined by Eq. (13–41). In calculating the computed values of *INS* observables from Eq. (13–131) in program Regres (as described in Section 13.7.1.2), the computed values of F_1, F_2, and F_3 doppler observables will correspond to the just-given definitions of these observables.

13.7.1.2 Computed Values, Media Corrections, and Partial Derivatives of Narrowband Spacecraft Interferometry (*INS*) Observables

13.7.1.2.1 Computed Values of Narrowband Spacecraft Interferometry (*INS*) Observables

Computed values of narrowband spacecraft interferometry (*INS*) observables are calculated from differenced computed doppler observables according to Eq. (13–131). Each computed doppler observable in this equation has the same time tag (TT) and count interval (T_c) as the observed value of the *INS* observable.

If the transmitter is the spacecraft, the definition of one-way doppler (F_1) observables in Eq. (13–131) is given by Eq. (13–41). Computed values of one-way doppler (F_1) observables defined by Eq. (13–41) are calculated from Eq. (13–82) as described in Section 13.3.2.3.

If the transmitter is a tracking station on Earth and the transmitter frequency $f_T(t_1)$ is constant, two-way doppler (F_2) and three-way doppler (F_3) in Eq. (13–131) are unramped doppler, which is defined by Eq. (13–31). Computed values of unramped two-way (F_2) and three-way (F_3) doppler observables defined by Eq. (13–31) are calculated from Eq. (13–47) as described in Section 13.3.2.1.

If the transmitter is a tracking station on Earth and the transmitter frequency $f_T(t_1)$ is ramped, two-way doppler (F_2) and three-way doppler (F_3) in Eq. (13–131) are ramped doppler, which is defined by Eq. (13–41). Computed values of ramped two-way (F_2) and three-way (F_3) doppler observables defined by Eq. (13–41) are calculated from Eq. (13–50) or Eq. (13–54) with the first term set equal to zero, as described in Section 13.3.2.2.1. The integral in the second term of either of these equations can be evaluated using ramp tables as described in Section 13.3.2.2.2 or phase tables as described in Section 13.3.2.2.3.

13.7.1.2.2 Media Corrections for Computed Values of Narrowband Spacecraft Interferometry (*INS*) Observables

From Eq. (13–131), the media correction ΔINS to the computed value INS of a narrowband spacecraft interferometry observable is the media correction ΔF_{i_2} to the doppler observable F_{i_2} received at tracking station 2 on Earth minus the media correction ΔF_{i_1} to the doppler observable F_{i_1} received at tracking station 1 on Earth. The subscript i is 1 for one-way doppler, 2 for two-way doppler, or 3 for three-way doppler. Also, round-trip doppler is unramped or ramped if the transmitter frequency is constant or ramped, respectively.

The media correction ΔF_1 to the computed value of a one-way doppler (F_1) observable is calculated in the Regres editor from Eqs. (13–85) and (13–86), as described in Section 13.3.2.3. In Eq. (13–85), $\Delta\rho_{1_e}$ and $\Delta\rho_{1_s}$ are media corrections to the precision one-way light times ρ_{1_e} and ρ_{1_s} calculated from the light-time solutions at the end and start of the doppler count interval T_c. These media corrections are calculated from Eqs. (10–24) and (10–25), as described in Section 10.2.

The media corrections ΔF_2 and ΔF_3 to the computed values of two-way (F_2) and three-way (F_3) doppler observables are calculated in the Regres editor from Eq. (13–48) for unramped doppler and Eq. (13–58) for ramped doppler. In these equations, $\Delta\rho_e$ and $\Delta\rho_s$ are media corrections to the precision round-trip light times ρ_e and ρ_s calculated from the light-time solutions at the end and start of the doppler count interval T_c. For doppler observables, these round-trip media corrections are calculated from Eqs. (10–28) and (10–29). However, in calculating media corrections for the computed values of *INS* observables from differenced doppler corrections, the up-leg corrections for the two doppler observables are almost identical, and their difference can be ignored. Hence, in Eqs. (13–48) and (13–58), the round-trip media corrections $\Delta\rho_e$ and $\Delta\rho_s$ are replaced with the down-leg corrections $\Delta\rho_{1_e}$ and $\Delta\rho_{1_s}$, which are calculated from Eqs. (10–24) and (10–25).

13.7.1.2.3 Partial Derivatives of Computed Values of Narrowband Spacecraft Interferometry (*INS*) Observables

From Eq. (13–131), the partial derivative $\partial INS/\partial \mathbf{q}$ of the computed value *INS* of a narrowband spacecraft interferometry observable with respect to the solve-for and consider parameter vector $\mathbf{q}$ is the partial derivative $\partial F_{i_2} \big/ \partial \mathbf{q}$ of the doppler observable F_{i_2} received at tracking station 2 on Earth with respect to $\mathbf{q}$ minus the partial derivative $\partial F_{i_1} \big/ \partial \mathbf{q}$ of the doppler observable F_{i_1} received at tracking station 1 on Earth with respect to $\mathbf{q}$. The subscript i is 1 for one-way doppler, 2 for two-way doppler, or 3 for three-way doppler. Also, round-trip doppler is unramped or ramped if the transmitter frequency is constant or ramped, respectively.

The partial derivative $\partial F_1/\partial \mathbf{q}$ of the computed value of a one-way doppler (F_1) observable with respect to the parameter vector $\mathbf{q}$ is calculated from Eqs. (13–87) to (13–90) as described in the accompanying text. In Eq. (13–87), the one-way light time partials are calculated from the formulation of Section 12.5.2.

The partial derivatives $\partial F_2/\partial \mathbf{q}$ and $\partial F_3/\partial \mathbf{q}$ of the computed values of two-way (F_2) and three-way (F_3) doppler observables with respect to the parameter vector $\mathbf{q}$ are calculated from Eq. (13–49) for unramped doppler and

Eq. (13–59) for ramped doppler, as described in the text accompanying these equations. In these equations, the round-trip light time partials are calculated from the formulation of Section 12.5.1.

13.7.2 WIDEBAND SPACECRAFT INTERFEROMETRY (*IWS*) OBSERVABLES

Section 13.7.2.1 describes the actual observed quantities and shows how these quantities are assembled to form the observed values of wideband spacecraft interferometry observables. It is also shown that a wideband spacecraft interferometry observable is equivalent to the difference of two spacecraft light times, which have the same reception time in station time ST at two different tracking stations on Earth. Wideband spacecraft interferometry observables are derived from two signals transmitted by the spacecraft. If these two signals are a fixed frequency apart at the spacecraft, the spacecraft light times are one-way light times. However, if the two signals transmitted at the spacecraft were derived from signals transmitted at a tracking station on Earth, which are a fixed frequency apart, then the spacecraft light times are round-trip (two-way or three-way) light times.

The formulation for the computed values of wideband spacecraft interferometry observables is given in Section 13.7.2.2.1. The formulations for the media corrections for the computed values of these observables and the partial derivatives of the computed values of these observables with respect to the solve-for and consider parameter vector **q** are given in Sections 13.7.2.2.2 and 13.7.2.2.3, respectively.

13.7.2.1 Observed Values of Wideband Spacecraft Interferometry (*IWS*) Observables

Subsection 13.7.2.1.1 gives the formulation used in the ODE to calculate the observed values of wideband spacecraft interferometry (*IWS*) observables. The corresponding definitions of one-way and round-trip *IWS* observables are developed in Subsections 13.7.2.1.2 and 13.7.2.1.3, respectively. These definitions are used in calculating the computed values of these observables in Section 13.7.2.2.1.

13.7.2.1.1 Formulation for Observed Values of *IWS* Observables

The phase difference $\left(\phi_2 - \phi_1\right)$ is defined near the beginning of Section 13.7.1.1. The observed quantities, which are used to construct the "observed" value of a wideband spacecraft interferometry observable, are measured values of $\left(\phi_2 - \phi_1\right)$ for each of two signals transmitted by the spacecraft. The frequencies of the two signals transmitted by the spacecraft are denoted as ω_B and ω_A, where $\omega_B > \omega_A$. Wideband spacecraft interferometry (*IWS*) observables are one-way if $\omega_B - \omega_A$, the difference in the frequencies of the two signals transmitted by the spacecraft, is constant at the spacecraft. This can occur if the spacecraft is the transmitter. However, if a tracking station on Earth is the transmitter and a single frequency is transmitted from the tracking station on Earth to the spacecraft, and ω_B and ω_A are functions of time, but $\omega_B - \omega_A$ at the spacecraft is constant, then the *IWS* observable is also one-way. If a tracking station on Earth transmits a single constant frequency to the spacecraft and $\omega_B - \omega_A$ at the spacecraft is not constant, the two signals transmitted at the spacecraft can be considered to be transmitted at the transmitting station on Earth and reflected off of the spacecraft. The difference in the frequencies of the two imaginary signals transmitted at the tracking station on Earth is a constant frequency, which is also denoted as $\omega_B - \omega_A$. For this case, where $\omega_B - \omega_A$ is constant at the transmitting station on Earth, the *IWS* observables are round-trip. If the transmitter is a tracking station on Earth and the transmitted frequency is ramped, then round-trip *IWS* observables cannot be taken.

Let the measured values of $\left(\phi_2 - \phi_1\right)$ for the transmitted frequencies ω_B and ω_A at the spacecraft or at a tracking station on Earth be denoted as $\left(\phi_2 - \phi_1\right)_B$ and $\left(\phi_2 - \phi_1\right)_A$, respectively. Given these measured quantities and the frequency difference $\omega_B - \omega_A$ at the spacecraft (one-way *IWS*) or at the transmitting station on Earth (round-trip *IWS*), the observed value of a one-way or round-trip *IWS* observable is calculated in the ODE from:

$$IWS = -\frac{\left[\left(\phi_2 - \phi_1\right)_{\mathrm{B}} - \left(\phi_2 - \phi_1\right)_{\mathrm{A}}\right]_{\text{fractional part}}}{\omega_{\mathrm{B}} - \omega_{\mathrm{A}}} \times 10^9 \qquad \text{ns} \qquad (13\text{–}132)$$

The "fractional part" of the numerator means that the integral part of the numerator is discarded. This is necessary to eliminate the constant errors of an integer number of cycles in each of the two measured phase differences. Since the numerator is in cycles and the denominator is in Hz, the quotient is in seconds. Multiplying by 10^9 gives the *IWS* observable in nanoseconds (ns). Calculating the fractional part of the numerator is equivalent to evaluating Eq. (13–132) without the fractional part calculation and then calculating the result modulo M, where M is given by:

$$M = \frac{10^9}{\omega_{\mathrm{B}} - \omega_{\mathrm{A}}} \qquad \text{ns} \qquad (13\text{–}133)$$

The value of the modulo number M will be passed to program Regres on the OD file along with the observed value of the *IWS* observable, given by Eq. (13–132). If Eq. (13–132) is evaluated in the ODE without the fractional part calculation, then the value of M passed to Regres will be zero. The time tag (TT) for the *IWS* observable is the common reception time $t_3(\mathrm{ST})_{\mathrm{R}}$ in station time ST at the receiving electronics at tracking stations 2 and 1 on Earth at which the phases in the numerator of Eq. (13–132) are measured.

13.7.2.1.2 Definition of One-Way *IWS* Observables

This section applies for the case where the frequency ω_{B} transmitted by the spacecraft minus the frequency ω_{A} transmitted by the spacecraft is a constant.

The phase difference in the numerator of Eq. (13–132) can be expressed as:

$$\left(\phi_2 - \phi_1\right)_{\mathrm{B}} - \left(\phi_2 - \phi_1\right)_{\mathrm{A}} = \left(\phi_{\mathrm{B}} - \phi_{\mathrm{A}}\right)_2 - \left(\phi_{\mathrm{B}} - \phi_{\mathrm{A}}\right)_1 \qquad \text{cycles} \qquad (13\text{–}134)$$

where

$\left(\phi_{\mathrm{B}} - \phi_{\mathrm{A}}\right)_i$ = difference in phase of the two signals received at the receiving electronics at tracking station i on Earth at the reception time $t_3(\mathrm{ST})_{\mathrm{R}}$ in station time ST. The two signals were transmitted at the spacecraft at frequencies ω_{B} and ω_{A}, respectively.

In the absence of charged particles (which will be considered separately in Section 13.7.2.2.2), the received phase difference $\left(\phi_{\mathrm{B}} - \phi_{\mathrm{A}}\right)_i$ at tracking station i on Earth is equal to the difference in phase of the two signals transmitted at the spacecraft at the transmission time $t_2(\mathrm{TAI})$ in International Atomic Time TAI at the spacecraft:

$$\left(\phi_{\mathrm{B}} - \phi_{\mathrm{A}}\right)_i = \left(\phi_{\mathrm{B}} - \phi_{\mathrm{A}}\right)_{t_2} \qquad \text{cycles} \qquad (13\text{–}135)$$

The phase difference at the spacecraft is a function of $t_2(\mathrm{TAI})$:

$$\left(\phi_{\mathrm{B}} - \phi_{\mathrm{A}}\right)_{t_2} = \left(\omega_{\mathrm{B}} - \omega_{\mathrm{A}}\right)\left[t_2(\mathrm{TAI}) - t_{2_0}(\mathrm{TAI})\right] \qquad \text{cycles} \qquad (13\text{–}136)$$

where $t_{2_0}(\mathrm{TAI})$ is the value of $t_2(\mathrm{TAI})$ at which the two signals transmitted at the spacecraft are in phase.

The definition of the precision one-way light time $\hat{\rho}_1$ from the spacecraft to a tracking station on Earth is given by Eq. (11–8). Substituting $t_2(\mathrm{TAI})$ from Eqs. (13–135) and (13–136) into Eq. (11–8) gives the following expression for the one-way light time $\hat{\rho}_1(i)$ received at tracking station i on Earth at $t_3(\mathrm{ST})_{\mathrm{R}}$ in station time ST at the receiving electronics:

$$\hat{\rho}_1(i) = t_3(\mathrm{ST})_\mathrm{R} - t_{2_0}(\mathrm{TAI}) - \frac{(\phi_\mathrm{B} - \phi_\mathrm{A})_i}{\omega_\mathrm{B} - \omega_\mathrm{A}} \qquad \mathrm{s} \qquad (13\text{–}137)$$

Let the differenced one-way light time $\Delta\hat{\rho}_1$ with the same reception time $t_3(\mathrm{ST})_\mathrm{R}$ in station time ST at the receiving electronics at tracking stations 2 and 1 on Earth be defined by:

$$\Delta\hat{\rho}_1 = \hat{\rho}_1(2) - \hat{\rho}_1(1) \qquad \mathrm{s} \qquad (13\text{–}138)$$

Substituting Eq. (13–137) with $i = 2$ and 1 into Eq. (13–138) and using Eq. (13–134) gives:

$$\Delta\hat{\rho}_1 = -\frac{\left[\left(\phi_2 - \phi_1\right)_\mathrm{B} - \left(\phi_2 - \phi_1\right)_\mathrm{A}\right]}{\omega_\mathrm{B} - \omega_\mathrm{A}} \qquad \mathrm{s} \qquad (13\text{–}139)$$

Comparing this equation to Eq. (13–132), we see that the definition of a one-way wideband spacecraft interferometry (*IWS*) observable is given by:

$$\text{one-way} \quad IWS = \Delta\hat{\rho}_1 \times 10^9, \text{ modulo } M \qquad \mathrm{ns} \qquad (13\text{–}140)$$

where the modulo number M is given by Eq. (13–133). In Eq. (13–140), $\Delta\hat{\rho}_1$ is given by Eq. (13–138), and the one-way light times at receiving stations 2 and 1 on Earth are each defined by Eq. (11–8). The definition (13–140) of a one-way wideband spacecraft interferometry (*IWS*) observable will be used in Section 13.7.2.2.1 to calculate the computed value of this observable.

13.7.2.1.3 Definition of Round-Trip *IWS* Observables

This section applies for the case where a constant frequency f_T is transmitted from a tracking station on Earth to the spacecraft. The spacecraft multiplies the received frequency $f_\mathrm{R}(t_2)$ by the spacecraft transponder turnaround ratio M_2 to give the downlink carrier frequency $f_\mathrm{T}(t_2)$. The spacecraft transponder produces harmonics by multiplying the downlink carrier frequency by a constant factor β and phase modulating the resulting signal onto the downlink carrier. The downlink signal contains the carrier of frequency $f_T(t_2)$ and harmonics that have frequencies equal to the downlink carrier frequency plus or minus integer multiples of the modulation frequency $f_T(t_2)\,\beta$. The two signals transmitted by the spacecraft that produce the observed phase differences $\left(\phi_2 - \phi_1\right)_B$ and $\left(\phi_2 - \phi_1\right)_A$ discussed in Section 13.7.2.1.1 are usually the upper and lower first harmonics.

Let $\phi_B(t_2)$ and $\phi_A(t_2)$ denote the phases of the upper and lower first harmonics transmitted by the spacecraft at the transmission time t_2 at the spacecraft. The corresponding frequencies of these two signals are $f_T(t_2)\,(1 + \beta)$ and $f_T(t_2)\,(1 - \beta)$, respectively. They are in phase at the time t_{2_0}. The phases $\phi_B(t_2)$ and $\phi_A(t_2)$ are given by:

$$\phi_{B,A}\left(t_2\right) = \int_{t_{2_0}}^{t_2} f_T\left(t_2\right)\left(1 \pm \beta\right) dt_2 \qquad \text{cycles} \qquad (13\text{–}141)$$

Replacing the downlink carrier frequency $f_T(t_2)$ with the uplink received frequency at the spacecraft multiplied by the spacecraft transponder turnaround ratio M_2 gives:

$$\phi_{B,A}\left(t_2\right) = M_2\,\left(1 \pm \beta\right) \int_{t_{2_0}}^{t_2} f_R\left(t_2\right) dt_2 \qquad \text{cycles} \qquad (13\text{–}142)$$

Since the up-leg signal travels at constant phase, this can be expressed as:

$$\phi_{B,A}\left(t_2\right) = M_2\,\left(1 \pm \beta\right) \int_{t_{1_0}\,(\mathrm{ST})_T}^{t_1\,(\mathrm{ST})_T} f_T\;dt_1 \qquad \text{cycles} \qquad (13\text{–}143)$$

where $t_1(\mathrm{ST})_\mathrm{T}$ and $t_{1_0}(\mathrm{ST})_\mathrm{T}$ are transmission times in station time ST at the transmitting electronics at the transmitting station on Earth. These times correspond to the reception times t_2 and t_{2_0} at the spacecraft. Since the transmitter frequency f_T at the transmitting station on Earth is constant, Eq. (13–143) reduces to:

$$\phi_{\mathrm{B,A}}(t_2) = M_2\, f_\mathrm{T}\,(1 \pm \beta)\left[t_1(\mathrm{ST})_\mathrm{T} - t_{1_0}(\mathrm{ST})_\mathrm{T}\right] \quad \text{cycles} \qquad (13\text{–}144)$$

In the absence of charged particles, whose effects are considered separately in Section 13.7.2.2.2, the difference in the phase of the two signals transmitted at the spacecraft at the transmission time t_2 is given by:

$$(\phi_\mathrm{B} - \phi_A)_{t_2} = (\omega_\mathrm{B} - \omega_\mathrm{A})\left[t_1(\mathrm{ST})_\mathrm{T} - t_{1_0}(\mathrm{ST})_\mathrm{T}\right] \quad \text{cycles} \qquad (13\text{–}145)$$

where

$$\omega_\mathrm{B} - \omega_\mathrm{A} = 2\, M_2\, f_\mathrm{T}\, \beta \quad \text{cycles} \qquad (13\text{–}146)$$

In the absence of charged particles, the received phase difference $(\phi_\mathrm{B} - \phi_\mathrm{A})_i$ at tracking station *i* on Earth is equal to the difference in phase $(\phi_\mathrm{B} - \phi_A)_{t_2}$ of the two signals transmitted at the spacecraft at the transmission time t_2 at the spacecraft, as shown in Eq. (13–135).

The definition of the precision round-trip light time ρ from a tracking station on Earth to the spacecraft and then to the same or a different tracking station on Earth is given by Eq. (11–5). Substituting $t_1(\mathrm{ST})_\mathrm{T}$ from Eqs. (13–135) and (13–145) into Eq. (11–5) gives the following expression for the round-trip light time $\rho(i)$ received at tracking station *i* on Earth at $t_3(\mathrm{ST})_\mathrm{R}$ in station time ST at the receiving electronics:

$$\rho(i) = t_3(\mathrm{ST})_\mathrm{R} - t_{1_0}(\mathrm{ST})_\mathrm{T} - \frac{(\phi_\mathrm{B} - \phi_\mathrm{A})_i}{\omega_\mathrm{B} - \omega_\mathrm{A}} \quad \text{s} \qquad (13\text{–}147)$$

where $\omega_B - \omega_A$ is given by Eq. (13–146). Let the differenced round-trip light time $\Delta\rho$ with the same reception time $t_3(\mathrm{ST})_R$ in station time ST at the receiving electronics at tracking stations 2 and 1 on Earth be defined by:

$$\Delta\rho = \rho(2) - \rho(1) \qquad \mathrm{s} \qquad (13\text{–}148)$$

Substituting Eq. (13–147) with $i = 2$ and 1 into Eq. (13–148) and using Eq. (13–134) gives:

$$\Delta\rho = -\frac{\left[\left(\phi_2 - \phi_1\right)_B - \left(\phi_2 - \phi_1\right)_A\right]}{\omega_B - \omega_A} \qquad \mathrm{s} \qquad (13\text{–}149)$$

Comparing this equation to Eq. (13–132), we see that the definition of a round-trip wideband spacecraft interferometry (*IWS*) observable is given by:

$$\text{round-trip } IWS = \Delta\rho \times 10^9, \text{ modulo } M \qquad \mathrm{ns} \qquad (13\text{–}150)$$

where the modulo number M is given by Eq. (13–133). In Eq. (13–150), $\Delta\rho$ is given by Eq. (13–148), and the round-trip light times at receiving stations 2 and 1 on Earth are each defined by Eq. (11–5). The definition (13–150) of a round-trip wideband spacecraft interferometry (*IWS*) observable will be used in Section 13.7.2.2.1 to calculate the computed value of this observable.

13.7.2.2 Computed Values, Media Corrections, and Partial Derivatives of Wideband Spacecraft Interferometry (*IWS*) Observables

13.7.2.2.1 Computed Values of Wideband Spacecraft Interferometry (*IWS*) Observables

If the frequency difference $\omega_B - \omega_A$ of the two signals transmitted by the spacecraft is constant at the spacecraft, wideband spacecraft interferometry (*IWS*) observables are one-way. However, if $\omega_B - \omega_A$ is constant at the transmitting station on Earth (as defined in Section 13.7.2.1.1), *IWS* observables are round-trip.

Computed values of one-way wideband spacecraft interferometry (*IWS*) observables are calculated from the definition equation (13–140). In this equation, the differenced one-way light time $\Delta\hat{\rho}_1$ is given by Eq. (13–138). In this equation, the one-way light times $\hat{\rho}_1(2)$ and $\hat{\rho}_1(1)$ with the same reception time $t_3(\mathrm{ST})_\mathrm{R}$ in station time ST at the receiving electronics at tracking stations 2 and 1 on Earth are defined by Eq. (11–8). The common reception time $t_3(\mathrm{ST})_\mathrm{R}$ is the time tag (TT) for the *IWS* observable.

The differenced one-way light time $\hat{\rho}_1(2) - \hat{\rho}_1(1)$ is calculated as the differenced one-way light time $\rho_1(2) - \rho_1(1)$ plus the variable Δ, as indicated in Eq. (11–11). The one-way light times $\rho_1(2)$ and $\rho_1(1)$ at receiving stations 2 and 1 on Earth are defined by Eq. (11–9). Given the one-way light-time solutions from the spacecraft to receiving stations 2 and 1 on Earth with the common reception time $t_3(\mathrm{ST})_\mathrm{R}$ at the receiving electronics at these two stations, the one-way light times $\rho_1(2)$ and $\rho_1(1)$ are calculated from Eq. (11–41). The parameter Δ, which is defined by Eq. (11–12), is calculated from Eqs. (11–15) to (11–39). These equations are evaluated with quantities obtained at the transmission times (t_2) of the two one-way spacecraft light-time solutions.

The differenced one-way light time $\hat{\rho}_1(2) - \hat{\rho}_1(1)$ is actually calculated in the code used to calculate the computed value of a one-way doppler observable. However, instead of calculating the differenced one-way light time at two different times (separated by the doppler count interval) at one tracking station on Earth, the differenced one-way light time is calculated at the same reception time $t_3(\mathrm{ST})_\mathrm{R}$ at two different tracking stations on Earth.

Computed values of round-trip wideband spacecraft interferometry (*IWS*) observables are calculated from the definition equation (13–150). In this equation, the differenced round-trip light time $\Delta\rho$ is given by Eq. (13–148). In this equation, the round-trip light times $\rho(2)$ and $\rho(1)$ with the same reception time $t_3(\mathrm{ST})_\mathrm{R}$ in station time ST at the receiving electronics at tracking stations 2 and 1 on Earth are defined by Eq. (11–5). The common reception time $t_3(\mathrm{ST})_\mathrm{R}$ is the time tag (TT) for the *IWS* observable.

Given the round-trip light-time solutions from the transmitting station on Earth to the spacecraft and from there to receiving stations 2 and 1 on Earth with the common reception time $t_3(\mathrm{ST})_\mathrm{R}$ at the receiving electronics at these two stations, the round-trip light times $\rho(2)$ and $\rho(1)$ at receiving stations 2 and 1 on Earth, which are defined by Eq. (11–5), are calculated from Eq. (11–7).

13.7.2.2.2 Media Corrections for Computed Values of Wideband Spacecraft Interferometry (*IWS*) Observables

One-way and round-trip wideband spacecraft interferometry (*IWS*) observables are defined by Eqs. (13–140) and (13–150), respectively. In these equations, the differenced one-way and round-trip light times are given by Eqs. (13–138) and (13–148), respectively. In terms of the observed phases at receiving station *i*, the one-way light time and the round-trip light time are given by Eqs. (13–137) and (13–147), respectively. Charged particles along the one-way or round-trip path to receiving station *i* affect the received phases ϕ_B and ϕ_A, which correspond to the transmitter frequencies ω_B and ω_A, respectively. From Eq. (13–135), the changes in the phases of the received signals at tracking station *i* on Earth are equal to the changes in the phases of the corresponding transmitted signals at the transmission time t_2 at the spacecraft. From Eq. (13–136), the changes in the phases of the two transmitted signals at the spacecraft for one-way *IWS* observables are given by:

$$\Delta\phi_\mathrm{B}(t_2) = \omega_\mathrm{B}\ \Delta t_{2_\mathrm{B}} \qquad \text{cycles} \qquad (13\text{–}151)$$

$$\Delta\phi_\mathrm{A}(t_2) = \omega_\mathrm{A}\ \Delta t_{2_\mathrm{A}} \qquad \text{cycles} \qquad (13\text{–}152)$$

From Eq. (13–145), the changes in the phases of the two transmitted signals at the spacecraft for round-trip *IWS* observables are given by:

$$\Delta\phi_\mathrm{B}(t_2) = \omega_\mathrm{B}\ \Delta t_{1_\mathrm{B}} \qquad \text{cycles} \qquad (13\text{–}153)$$

$$\Delta\phi_\mathrm{A}(t_2) = \omega_\mathrm{A}\ \Delta t_{1_\mathrm{A}} \qquad \text{cycles} \qquad (13\text{–}154)$$

For one-way *IWS* observables, the changes in the transmission times at the spacecraft for the signals transmitted at the frequencies ω_B and ω_A, respectively, are:

$$\Delta t_{2_B} = \frac{C_2}{\omega_B^{\,2}} \qquad \text{s} \qquad (13\text{–}155)$$

$$\Delta t_{2_A} = \frac{C_2}{\omega_A^{\,2}} \qquad \text{s} \qquad (13\text{–}156)$$

where the constant C_2 is a function of the electron content along the down-leg light path. For round-trip *IWS* observables, the changes in the transmission times at the transmitting station on Earth for the signals transmitted at the spacecraft at the frequencies ω_B and ω_A, respectively, are:

$$\Delta t_{1_B} = \frac{C_2}{\omega_B^{\,2}} + \frac{C_1}{f_T^{\,2}} \qquad \text{s} \qquad (13\text{–}157)$$

$$\Delta t_{1_A} = \frac{C_2}{\omega_A^{\,2}} + \frac{C_1}{f_T^{\,2}} \qquad \text{s} \qquad (13\text{–}158)$$

where the constant C_1 is a function of the electron content along the up-leg light path and f_T is the up-leg transmitter frequency. The change in the one-way light time from the spacecraft to receiving station *i* on Earth due to charged particles is obtained by substituting Eqs. (13–151), (13–152), (13–155), and (13–156) into the differentials of Eqs. (13–137) and (13–135):

$$\Delta\hat{\rho}_1(i) = \frac{C_2}{\omega_A\,\omega_B} \approx \frac{C_2}{\left(\dfrac{\omega_A + \omega_B}{2}\right)^2} \qquad \text{s} \qquad (13\text{–}159)$$

where the approximate form is the increase in the light time due to propagation at the group velocity (less than the speed of light *c*) for the average frequency $(\omega_A + \omega_B)/2$. The change in the round-trip light time from the transmitting station on Earth to the spacecraft and then to receiving station i on Earth due to charged particles is obtained by substituting Eqs. (13–153), (13–154), (13–157), and (13–158) into the differentials of Eqs. (13–147) and (13–135):

$$\Delta\rho(i) = \frac{C_2}{\omega_A \omega_B} - \frac{C_1}{{f_T}^2} \qquad \text{s} \qquad (13\text{–}160)$$

The first term is the same as Eq. (13–159). The second term is the decrease in the light time due to propagation on the up leg at the phase velocity (greater than the speed of light c) for the transmitter frequency f_T. The up-leg correction is a phase velocity correction instead of the usual group velocity correction for a range observable because only one signal is transmitted on the up leg. From Eqs. (13–150) and (13–148), the media correction to the computed value of a round-trip wideband spacecraft interferometry observable is proportional to the media correction to the round-trip light time $\rho(2)$ received at tracking station 2 on Earth minus the media correction to the round-trip light time $\rho(1)$ received at tracking station 1 on Earth, where both light times have the same reception time $t_3(\mathrm{ST})_R$ at the receiving electronics at stations 2 and 1. In Eq. (13–160) for the charged-particle correction for the round-trip light time, the down-leg corrections to receiving stations 2 and 1 on Earth are different. However, the up-leg charged-particle corrections (and the up-leg tropospheric corrections) are nearly the same. They differ only because the transmission times t_2 at the spacecraft for the two different receiving stations on Earth differ by the difference in the two down-leg light times. The difference in the up-leg media corrections for receiving stations 2 and 1 can be ignored and Eq. (13–160) reduces to its first term, which is the same as Eq. (13–159).

From the above, the media correction for the computed value of a one-way or round-trip wideband spacecraft interferometry (*IWS*) observable is calculated in the Regres editor from:

$$\Delta IWS = \left[\Delta\rho_1(2) - \Delta\rho_1(1)\right] \times 10^9 \qquad \text{ns} \qquad (13\text{–}161)$$

where $\Delta\rho_1(2)$ and $\Delta\rho_1(1)$ are down-leg media corrections in seconds for the down-leg light times $\rho_1(2)$ and $\rho_1(1)$, which have reception times $t_3(\mathrm{ST})_\mathrm{R}$ at the receiving electronics at receiving stations 2 and 1 on Earth, respectively. The down-leg light times $\rho_1(2)$ and $\rho_1(1)$ are the down-leg light times on the right-hand side of Eq. (11–11), which are defined by Eq. (11–9) or the down-leg terms of the round-trip light times defined by Eq. (11–5). The one-way media corrections $\Delta\rho_1(2)$ and $\Delta\rho_1(1)$ (tropospheric plus charged-particle corrections) are calculated in the Regres editor from Eq. (10–26) as described in Section 10.2. The charged-particle corrections are positive and based upon the average spacecraft transmitter frequency $\left(\omega_\mathrm{A} + \omega_\mathrm{B}\right)/2$. To sufficient accuracy, this frequency is given by $C_2 f_{\mathrm{T}_0}$ (see Eq. 13–21) if the spacecraft is the transmitter and by $M_2 f_\mathrm{T}$ (see Eq. 13–20) if a tracking station on Earth is the transmitter.

13.7.2.2.3 Partial Derivatives of Computed Values of Wideband Spacecraft Interferometry (*IWS*) Observables

From Eqs. (13–140), (13–138), and (11–11), the partial derivative of the computed value of a one-way wideband spacecraft interferometry observable with respect to the solve-for and consider parameter vector **q** is given by:

$$\frac{\partial\left(\text{one - way } IWS\right)}{\partial \mathbf{q}} = \left[\frac{\partial \rho_1(2)}{\partial \mathbf{q}} - \frac{\partial \rho_1(1)}{\partial \mathbf{q}}\right] \times 10^9 \qquad (13\text{–}162)$$

where the one-way light times $\rho_1(2)$ and $\rho_1(1)$, which have the common reception time $t_3(\mathrm{ST})_\mathrm{R}$ at the receiving electronics at receiving stations 2 and 1 on Earth, are defined by Eq. (11–9). The partial derivatives of these one-way light times with respect to the parameter vector **q** are calculated from the formulation of Section 12.5.2.

The differenced one-way light time partial derivatives in Eq. (13–162) are calculated in the code used to calculate the computed values of one-way doppler observables and partial derivatives, as described in Section 13.7.2.2.1.

From Eqs. (13–150) and (13–148), the partial derivative of the computed value of a round-trip wideband spacecraft interferometry observable with respect to the parameter vector **q** is given by:

$$\frac{\partial\left(\text{round - trip } IWS\right)}{\partial \mathbf{q}} = \left[\frac{\partial \rho(2)}{\partial \mathbf{q}} - \frac{\partial \rho(1)}{\partial \mathbf{q}}\right] \times 10^{9} \qquad (13\text{–}163)$$

where the round-trip light times $\rho(2)$ and $\rho(1)$, which have the common reception time $t_3(\mathrm{ST})_\mathrm{R}$ at the receiving electronics at receiving stations 2 and 1 on Earth, are defined by Eq. (11–5). The partial derivatives of these round-trip light times with respect to the parameter vector **q** are calculated from the formulation of Section 12.5.1.

13.8 QUASAR INTERFEROMETRY OBSERVABLES

This section gives the formulation for the observed and computed values of narrowband (*INQ*) and wideband (*IWQ*) quasar interferometry observables, media corrections for the computed observables, and partial derivatives of the computed values of the observables with respect to the solve-for and consider parameter vector **q**. These data types are only processed in the Solar-System barycentric space-time frame of reference.

Section 13.8.1.1 describes the actual observed quantities. Sections 13.8.1.2 and 13.8.1.3 show how these observed quantities are assembled to form the observed values of narrowband and wideband quasar interferometry observables, respectively. These two sections also give the definitions of narrowband and wideband quasar interferometry observables. These definitions are used in calculating the computed values of these observables in Sections 13.8.2.1 and 13.8.2.2. The formulations for the media corrections for the computed values of these observables and the partial derivatives of the computed values of these observables with respect to the solve-for and consider parameter vector **q** are given in Sections 13.8.2.3 and 13.8.2.4, respectively.

13.8.1 OBSERVED VALUES OF QUASAR INTERFEROMETRY OBSERVABLES

13.8.1.1 Observed Quantities

The signal from a quasar is received on a given channel at receiver 1 and at receiver 2. Each of these two receivers can be a tracking station on Earth or an Earth satellite. Correlation of the quasar signals received on a given channel at these two receivers gives a continuous phase ϕ vs station time ST at the receiving electronics at receiver 1. The phase ϕ is defined by:

$$n + \phi = \overline{\omega}\,\tau \qquad \text{cycles} \qquad (13\text{–}164)$$

$$\tau = t_2(\mathrm{ST})_\mathrm{R} - t_1(\mathrm{ST})_\mathrm{R} \qquad \text{s} \qquad (13\text{–}165)$$

where

$\overline{\omega}$ = effective frequency of quasar for a specific channel and pass (Hz).

$t_2(\mathrm{ST})_\mathrm{R}$ = reception time of quasar wavefront at receiver 2 in station time ST at receiving electronics at receiver 2.

$t_1(\mathrm{ST})_\mathrm{R}$ = reception time of quasar wavefront at receiver 1 in station time ST at receiving electronics at receiver 1.

n = an unknown integer, whose value is typically a few tens of cycles (constant during a pass).

Note that Eq. (13–165) is the same as Eq. (11–65). Except for the error n, the phase ϕ is the phase of the received quasar signal at $t_1(\mathrm{ST})_\mathrm{R}$ in station time ST at the receiving electronics at receiver 1 minus the phase of the received quasar signal at the same value of station time ST at the receiving electronics at receiver 2. Note that the phase of the received quasar signal at $t_2(\mathrm{ST})_\mathrm{R}$ in station time ST at the receiving electronics at receiver 2 is equal to the phase of the received quasar signal at $t_1(\mathrm{ST})_\mathrm{R}$ in station time ST at the receiving electronics at receiver 1. Except for the error n, the phase ϕ is the number of cycles (or $\phi/\overline{\omega}$ seconds of station time ST) that the received waveform vs station time at station 2 must be moved backward in time to line up with the received waveform vs station time at station 1.

For narrowband quasar interferometry (*INQ*), the phase ϕ vs station time ST at the receiving electronics at receiver 1 is available from one channel only. For wideband quasar interferometry (*IWQ*), the phase ϕ vs station time ST at the receiving electronics at receiver 1 is available from two channels.

The equations and definitions given above are valid for a positive delay τ and phase ϕ (the quasar wavefront arrives at receiver 1 first) and also for a negative delay τ and phase ϕ (the quasar wavefront arrives at receiver 2 first).

If receiver 1 or receiver 2 is an Earth satellite, the orbit of that satellite can be determined by fitting to quasar interferometry data and other tracking data.

For a deep space probe, the trajectory of the spacecraft can be determined by fitting to $\Delta VLBI$ as described in the last paragraph of Section 13.7.

13.8.1.2 Formulation for Observed Values and Definition of *INQ* Observables

The observed value of a narrowband quasar interferometry (*INQ*) observable is calculated in the ODE from:

$$INQ = \frac{\phi_e - \phi_s}{T_c} \qquad \text{Hz} \qquad (13\text{–}166)$$

where

ϕ_e = value of the phase ϕ defined by Eqs. (13–164) and (13–165) and the accompanying text with a reception time $t_1(\mathrm{ST})_R$ in station time ST at the receiving electronics at receiver 1 equal to the data time tag TT plus one-half of the count interval T_c.

ϕ_s = value of the phase ϕ defined by Eqs. (13–164) and (13–165) and the accompanying text with a reception time $t_1(\mathrm{ST})_R$ in station time ST at the receiving electronics at receiver 1 equal to the data time tag TT minus one-half of the count interval T_c.

T_c = count interval in seconds of station time ST at the receiving electronics at receiver 1. It is equal to the reception time $t_1(\mathrm{ST})_R$ at receiver 1 for ϕ_e minus the reception time $t_1(\mathrm{ST})_R$ at receiver 1 for ϕ_s. For a pass of *INQ* data, the configuration of the count intervals can be in the doppler mode as shown in Figure 13–1 or in the phase mode as shown in Figure 13–2.

From the definition of the phase ϕ given in this section and the definition of the phase difference $(\phi_2 - \phi_1)$ given in Section 13.7.1.1, it is obvious that the latter is the negative of the former. Hence, Eq. (13–166) for the observed value of a narrowband quasar interferometry observable is equal to Eq. (13–130) for the observed value of a narrowband spacecraft interferometry observable. The quasar and spacecraft observables are calculated from the equivalent measured phases using the same equation. The only difference is the source of the signals that produce the measured phase differences.

From Eqs. (13–166) and (13–164), the definition of a narrowband quasar interferometry observable is given by:

$$INQ = \frac{\overline{\omega}\left(\tau_e - \tau_s\right)}{T_c} \qquad \text{Hz} \qquad (13\text{–}167)$$

where

τ_e , τ_s = quasar delays defined by Eq. (11–65) or Eq. (13–165) with reception times in station time ST at the receiving electronics at receiver 1 equal to the data time tag TT plus $T_c/2$ and TT minus $T_c/2$, respectively.

13.8.1.3 Formulation for Observed Values and Definition of *IWQ* Observables

The observed value of a wideband quasar interferometry (*IWQ*) observable can be calculated modulo *M* nanoseconds, or the observable can be unmodded. If the observed value of an *IWQ* observable is modded, it is calculated in the ODE from:

$$IWQ = \frac{\left(\phi_B - \phi_A\right)_{\text{fractional part}}}{\overline{\omega}_B - \overline{\omega}_A} \times 10^9 \qquad \text{ns} \qquad (13\text{–}168)$$

where

ϕ_B , ϕ_A = values of the phase ϕ defined by Eqs. (13–164) and (13–165) and the accompanying text for channels B and

A. Each of these phases has the same reception time $t_1(\mathrm{ST})_\mathrm{R}$ in station time ST at the receiving electronics at receiver 1. This reception time is the time tag (TT) for the data point.

$\overline{\omega}_\mathrm{B}, \overline{\omega}_\mathrm{A}$ = values of $\overline{\omega}$ for channels B and A, where $\overline{\omega}_\mathrm{B} > \overline{\omega}_\mathrm{A}$.

The "fractional part" of the numerator means that the integral part of the numerator is discarded. This is necessary to eliminate the constant errors of an integer number of cycles in each of the two measured phases. Since the numerator is in cycles and the denominator is in Hz, the quotient is in seconds. Multiplying by 10^9 gives the *IWQ* observable in nanoseconds (ns). Calculating the fractional part of the numerator is equivalent to evaluating Eq. (13–168) without the fractional part calculation and then calculating the result modulo M, where M is given by:

$$M = \frac{10^9}{\overline{\omega}_\mathrm{B} - \overline{\omega}_\mathrm{A}} \qquad \mathrm{ns} \qquad (13\text{–}169)$$

The value of the modulo number M is passed to program Regres on the OD file along with the observed value of the *IWQ* observable. If the *IWQ* observable is not modded (as discussed below), then the value of M passed to Regres is zero.

From Eqs. (13–168) and (13–164), the definition of a wideband quasar interferometry observable which is calculated modulo M is given by:

$$IWQ = \tau \times 10^9, \ \text{modulo}\ M \qquad \mathrm{ns} \qquad (13\text{–}170)$$

where the quasar delay τ is defined by Eq. (11–65) or Eq. (13–165). It has a reception time $t_1(\mathrm{ST})_\mathrm{R}$ in station time ST at the receiving electronics at receiver 1 equal to the time tag TT for the data point.

If the observed value of an *IWQ* observable is unmodded, it is calculated in the ODE from the following variation of Eq. (13–168):

$$IWQ = \frac{\left(\phi_B - \phi_A\right)_{\text{fractional part}} + N}{\bar{\omega}_B - \bar{\omega}_A} \times 10^9 \qquad \text{ns} \qquad (13\text{–}171)$$

where the integer N is calculated from:

$$N = \left[\left(\bar{\omega}_B - \bar{\omega}_A\right)\tau_m\right]_{\text{integral part}} \qquad \text{cycles} \qquad (13\text{–}172)$$

where τ_m is the modelled delay in seconds used in the correlation process for either channel. The value of the integer N will occasionally be in error by plus or minus one cycle. In order to check the value of N, we need an observed value of the quasar delay τ in seconds. It is given by Eq. (13–171) without the factor 10^9. If the value of τ in seconds satisfies the inequality:

$$\left|\tau - \tau_m\right| << \frac{1 \text{ cycle}}{\bar{\omega}_B - \bar{\omega}_A} \qquad \text{s} \qquad (13\text{–}173)$$

N is presumed to be correct. If the inequality is not satisfied, calculate two new values of τ from Eq. (13–171) divided by 10^9 using $N + 1$ cycles and $N - 1$ cycles. If either of these values of N satisfies the inequality, that value of N should be used to calculate the IWQ observable from Eq. (13–171). Otherwise, delete the data point.

From Eqs. (13–171), (13–172), and (13–164), the definition of a wideband quasar interferometry observable which is not calculated modulo M is given by:

$$IWQ = \tau \times 10^9 \qquad \text{ns} \qquad (13\text{–}174)$$

where the quasar delay τ is defined by Eq. (11–65) or Eq. (13–165). It has a reception time $t_1(\text{ST})_R$ in station time ST at the receiving electronics at receiver 1 equal to the time tag TT for the data point.

13.8.2 COMPUTED VALUES, MEDIA CORRECTIONS, AND PARTIAL DERIVATIVES OF QUASAR INTERFEROMETRY OBSERVABLES

13.8.2.1 Computed Values of Narrowband Quasar Interferometry *INQ* Observables

The quasar light-time solution (Section 8.4.3) starts with the reception time $t_1(\mathrm{ST})_\mathrm{R}$ of the quasar wavefront in station time ST at the receiving electronics at receiver 1 and produces the reception time $t_2(\mathrm{ST})_\mathrm{R}$ of the quasar wavefront in station time ST at the receiving electronics at receiver 2. Given the quasar light-time solution, the precision quasar delay τ, which is defined by Eq. (11–65), is calculated from Eq. (11–67).

In order to calculate the computed value of a narrowband quasar interferometry (*INQ*) observable, quasar light-time solutions are performed at the end and at the start of the count interval T_c. The reception times of the quasar wavefront in station time ST at the receiving electronics at receiver 1 at the end and start of the count interval are given by the data time tag TT plus and minus one-half the count interval T_c, respectively (see Eqs. 10–37 with a subscript R added to each reception time). Given the two quasar light-time solutions, the precision quasar delay τ_e at the end of the count interval and the precision quasar delay τ_s at the start of the count interval are calculated from Eq. (11–67). Given these two quasar delays, the computed value of an *INQ* observable is calculated from Eq. (13–167), where the effective frequency $\overline{\omega}$ of the quasar and the count interval T_c are obtained from the data record for the data point on the OD file.

13.8.2.2 Computed Values of Wideband Quasar Interferometry *IWQ* Observables

In order to calculate the computed value of a wideband quasar interferometry (*IWQ*) observable, one quasar light-time solution is performed. The reception time of the quasar wavefront in station time ST at the receiving electronics at receiver 1 is given by the data time tag TT (see Eq. 10–36 with a subscript R added to the reception time). Given the quasar light-time solution, the precision quasar delay τ is calculated from Eq. (11–67). The modulo number

M for the data point, which is given by Eq. (13–169), is obtained from the data record for the data point on the OD file. If $M > 0$, the computed value of the *IWQ* observable is calculated from Eq. (13–170). However, if $M = 0$, the computed value of the *IWQ* observable is calculated from Eq. (13–174).

13.8.2.3 Media Corrections for Computed Values of Quasar Interferometry Observables

From Eq. (13–167), the media correction for the computed value of a narrowband quasar interferometry (*INQ*) observable is calculated in the Regres editor from:

$$\Delta INQ = \frac{\overline{\omega}\left(\Delta\tau_{\mathrm{e}} - \Delta\tau_{\mathrm{s}}\right)}{T_{\mathrm{c}}} \qquad \text{Hz} \qquad (13\text{–}175)$$

where $\Delta\tau_{\mathrm{e}}$ and $\Delta\tau_{\mathrm{s}}$ are media corrections to the quasar delays τ_{e} and τ_{s} at the end and start of the count interval T_{c}. The media corrections $\Delta\tau_{\mathrm{e}}$ and $\Delta\tau_{\mathrm{s}}$ are calculated from Eqs. (10–31) and (10–32) as described in Section 10.2. In these equations, the charged-particle corrections at receiver 2 and at receiver 1 are negative, which corresponds to propagation at the phase velocity for the effective quasar frequency $\overline{\omega}$. If a receiver is an Earth satellite, the troposphere and charged particle corrections for that receiver are zero.

From Eq. (13–168) or (13–171), the media correction for the computed value of a wideband quasar interferometry (*IWQ*) observable is given by:

$$\Delta IWQ = \frac{\Delta\phi_{\mathrm{B}} - \Delta\phi_{\mathrm{A}}}{\overline{\omega}_{\mathrm{B}} - \overline{\omega}_{\mathrm{A}}} \times 10^{9} \qquad \text{ns} \qquad (13\text{–}176)$$

where $\Delta\phi_{\mathrm{B}}$ and $\Delta\phi_{\mathrm{A}}$ are media corrections to the measured phases ϕ_{B} and ϕ_{A}, which are defined after Eq. (13–168). From Eq. (13–164), these phase corrections are given by:

$$\Delta\phi_{\mathrm{B}} = \overline{\omega}_{\mathrm{B}}\,\Delta\tau_{\mathrm{B}} \qquad \text{cycles} \qquad (13\text{–}177)$$

$$\Delta\phi_{\mathrm{A}} = \overline{\omega}_{\mathrm{A}}\,\Delta\tau_{\mathrm{A}} \qquad \text{cycles} \qquad (13\text{–}178)$$

where $\Delta\tau_{\mathrm{B}}$ and $\Delta\tau_{\mathrm{A}}$ are media corrections to the quasar delay τ for effective quasar frequencies $\overline{\omega}_{\mathrm{B}}$ and $\overline{\omega}_{\mathrm{A}}$, respectively. The troposphere corrections are not frequency dependent and produce troposphere corrections for τ in Eq. (13–170) or (13–174). We only need to consider the charged-particle corrections here. The charged-particle corrections for $\Delta\tau_{\mathrm{B}}$ and $\Delta\tau_{\mathrm{A}}$ are given by:

$$\Delta\tau_{\mathrm{B}} = -\left(\frac{C_2}{\overline{\omega}_{\mathrm{B}}^{\,2}} - \frac{C_1}{\overline{\omega}_{\mathrm{B}}^{\,2}}\right) \qquad \text{s} \qquad (13\text{–}179)$$

$$\Delta\tau_{\mathrm{A}} = -\left(\frac{C_2}{\overline{\omega}_{\mathrm{A}}^{\,2}} - \frac{C_1}{\overline{\omega}_{\mathrm{A}}^{\,2}}\right) \qquad \text{s} \qquad (13\text{–}180)$$

where the constants C_2 and C_1 are functions of the electron content along the down-leg light paths to receivers 2 and 1, respectively. The first and second terms in these equations are the charged-particle corrections along the down-leg light paths to receivers 2 and 1, respectively. The lead negative sign appears because the signals to each station propagate at the phase velocity, which is greater than the speed of light c. Substituting Eqs. (13–177) to (13–180) into Eq. (13–176) gives the charged-particle contribution to the media correction for a wideband quasar interferometry (*IWQ*) observable:

$$\Delta IWQ\ (\text{charged particles}) = \frac{C_2 - C_1}{\overline{\omega}_{\mathrm{A}}\,\overline{\omega}_{\mathrm{B}}} \times 10^9 \approx \frac{C_2 - C_1}{\left(\dfrac{\overline{\omega}_{\mathrm{A}} + \overline{\omega}_{\mathrm{B}}}{2}\right)^2} \times 10^9 \qquad \text{ns} \qquad (13\text{–}181)$$

The approximate form of this equation is the charged-particle correction on the down leg to receiver 2 minus the corresponding correction for receiver 1, where each correction is the increase in the light time due to propagation at the group velocity (less than the speed of light c) for the average frequency $(\overline{\omega}_{\mathrm{A}} + \overline{\omega}_{\mathrm{B}})/2$. This correction is the same as the group-velocity charged-particle correction for Eq. (13–170) or (13–174).

From the above, the media correction for the computed value of a wideband quasar interferometry (*IWQ*) observable given by Eq. (13–170) or (13–174) is calculated in the Regres editor from:

$$\Delta IWQ = \Delta\tau \times 10^9 \qquad \text{ns} \tag{13–182}$$

where the media correction for the quasar delay τ is calculated from Eq. (10–30) as described in Section 10.2. The charged-particle corrections along the light paths to receivers 2 and 1 are positive, and correspond to propagation at the group velocity for the average frequency $\left(\overline{\omega}_{\mathrm{A}} + \overline{\omega}_{\mathrm{B}}\right)/2$, which is obtained from the data record for the data point on the OD file. If a receiver is an Earth satellite, the troposphere and charged-particle corrections for that receiver are zero.

13.8.2.4 Partial Derivatives of Computed Values of Quasar Interferometry Observables

From Eq. (13–167), the partial derivative of the computed value of a narrowband quasar interferometry (*INQ*) observable with respect to the solve-for and consider parameter vector **q** is given by:

$$\frac{\partial INQ}{\partial \mathbf{q}} = \frac{\overline{\omega}}{T_{\mathrm{c}}}\left[\frac{\partial \tau_{\mathrm{e}}}{\partial \mathbf{q}} - \frac{\partial \tau_{\mathrm{s}}}{\partial \mathbf{q}}\right] \tag{13–183}$$

From Eq. (13–170) or (13–174), the partial derivative of the computed value of a wideband quasar interferometry (*IWQ*) observable with respect to the solve-for and consider parameter vector **q** is given by:

$$\frac{\partial IWQ}{\partial \mathbf{q}} = \frac{\partial \tau}{\partial \mathbf{q}} \times 10^9 \tag{13–184}$$

The partial derivatives of the quasar delay τ in Eq. (13–184) and the delays τ_{e} and τ_{s} at the end and start of the count interval in Eq. (13–183) with respect to the solve-for and consider parameter vector **q** are calculated from the formulation of Section 12.5.4 as described in that section.

13.9 ANGULAR OBSERVABLES

This section specifies the formulation for the computed values of angular observables and the partial derivatives of the computed values of angular observables with respect to the solve-for and consider parameter vector **q**. Angular observables are measured on the down-leg light path from a free or a landed spacecraft to a tracking station on Earth at the reception time t_3 at the tracking station. Observed angles are measured in pairs (*e.g.*, azimuth and elevation angles). The formulation for the computed values of angular observables is given in Section 9. This formulation is also used to calculate auxiliary angles. Auxiliary angles are calculated at the reception time at the receiving station on Earth and for round-trip light-time solutions at the corresponding transmission time t_1 at the transmitting station on Earth. Auxiliary angles are also calculated at the transmitting GPS satellite and at the receiving TOPEX satellite. For quasar data types, the auxiliary angles are the angular coordinates of the transmitting quasar. Computed values of angular observables are corrected for atmospheric refraction. In general, auxiliary angles are not corrected for refraction (see Section 9.3.1 for details).

Section 13.9.1 refers to Section 9 and summarizes how the various parts of this formulation are used to calculate the computed values of angular observables. The formulation for the partial derivatives of computed values of angular observables with respect to the parameter vector **q** is given in Section 13.9.2.

13.9.1 COMPUTED VALUES OF ANGULAR OBSERVABLES

The formulation for the computed values of angular observables is given in Sections 9.1 through 9.4. Figures 9–1, and 9–3 to 9–5 show the angle pairs: hour angle (HA) and declination (δ), azimuth (σ) and elevation (γ), X and Y, and X' and Y', respectively. Each of these figures shows the coordinate system to which the angle pair is referred and unit vectors in the directions of increases in these angles. The HA-δ angle pair plus the east longitude λ of the tracking station on Earth are referred to the Earth-fixed rectangular coordinate system aligned with the Earth's true pole, prime meridian, and true equator of date. The remaining three angle pairs are referred to the north-east-zenith coordinate system at the tracking station, which is shown in Figure 9–2. The unit vectors **N**, **E**, and **Z**, with rectangular components referred to the above-referenced Earth-fixed rectangular coordinate system are calculated from Eqs. (9–3) to (9–8). The unit vectors shown in Fig 9–1 are calculated from the computed values of the angular observables and the tracking station longitude shown in that figure. The unit vectors shown in Figs. 9–3 to 9–5 are calculated from the computed values of the angular observables and the unit vectors **N**, **E**, and **Z.** All of the unit vectors in the directions of increases in the angular observables are referred to the Earth-fixed true rectangular coordinate system. Eq. (9–15) is used to transform these Earth-fixed unit vectors to the corresponding space-fixed unit vectors referred to the celestial reference frame of the planetary ephemeris (see Section 3.1.1). The unit vector $\tilde{\mathbf{D}}$ in the direction of increasing elevation angle γ is used in calculating the refraction correction. All of the unit vectors are used in calculating the partial derivatives of the computed values of the angular observables with respect to the parameter vector **q**.

Calculation of the computed values of a pair of angular observables requires the unit vector **L** directed outward along the incoming raypath at the receiving station on Earth. Given the spacecraft light-time solution, the space-fixed unit vector **L**, which has rectangular components referred to the space-fixed coordinate system of the planetary ephemeris (nominally aligned with the mean Earth equator and equinox of J2000) is calculated from Eqs. (9–16), (9–19), and (9–21). The vector **L** includes the aberration correction calculated from Eq. (9–19). Eq. (9–22) transforms **L** from space-fixed to Earth-fixed components, which are referred to the Earth's true pole, prime meridian, and true equator of date. Eq. (9–23) corrects **L** for atmospheric refraction. The refraction correction $\Delta_{\mathrm{r}}\gamma$, which is the increase in the elevation angle γ due to atmospheric refraction, is calculated from the modified Berman-Rockwell model as specified in Section 9.3.2.1 or the Lanyi model as specified in Section 9.3.2.2. Given the Earth-fixed refracted unit vector **L**, computed values of hour angle (HA) and declination (δ) are calculated from Eqs. (9–38) to (9–41). Given this **L** and the Earth-fixed unit vectors **N**, **E**, and

Z, computed values of azimuth (σ) and elevation (γ), X and Y, and X' and Y' are calculated from Eqs. (9–42) to (9–48).

Section 9.4 gives equations for differential corrections to the computed values of angular observables due to small solve-for rotations of the reference coordinate system at the tracking station about each of its three mutually perpendicular axes. Figure 9–1 shows rotations of the reference coordinate system **PEQ** for hour angle (HA) and declination (δ) angles through the small angles η' about **P**, ε about **E**, and ζ' about **Q**. Figures 9–3 to 9–5 show rotations of the reference coordinate system **NEZ** for azimuth (σ) and elevation (γ), X and Y, and X' and Y' angles through the small angles η about **N**, ε about **E**, and ζ about **Z**. The differential corrections for the computed values of the angular observables are functions of the computed values of the angular observables and the solve-for rotations.

The tracking station coordinates used to calculate the computed values of angular observables should be corrected for polar motion. The maximum effect of polar motion on the computed values of angular observables is less than 0.0002 degree. From Section 9.2, the accuracy of angular observables is about 0.001 degree. Hence, the geocentric latitude ϕ and east longitude λ of a tracking station on Earth, which are used to calculate computed values of angular observables at that station, are not corrected for polar motion.

13.9.2 PARTIAL DERIVATIVES OF COMPUTED VALUES OF ANGULAR OBSERVABLES

Subsection 13.9.2.1 gives the high-level equations for the partial derivatives of computed values of angular observables with respect to the solve-for and consider parameter vector **q**. The required sub partial derivatives of the computed values of angular observables with respect to the position vectors of the receiver and transmitter are developed in Subsection 13.9.2.2.

13.9.2.1 High-Level Equations

For most parameters, the partial derivative of the computed value z of an angular observable with respect to the parameter vector $\mathbf{q}$ is calculated from:

$$\frac{\partial z}{\partial \mathbf{q}} = \frac{\partial z}{\partial \mathbf{r}_3^{\mathrm{C}}(t_3)} \frac{\partial \mathbf{r}_3^{\mathrm{C}}(t_3)}{\partial \mathbf{q}} + \frac{\partial z}{\partial \mathbf{r}_2^{\mathrm{C}}(t_2)} \left[\frac{\partial \mathbf{r}_2^{\mathrm{C}}(t_2)}{\partial \mathbf{q}} + \dot{\mathbf{r}}_2^{\mathrm{C}}(t_2) \frac{\partial t_2(\mathrm{ET})}{\partial \mathbf{q}} \right] \tag{13–185}$$

If the spacecraft light-time solution is performed in the Solar-System barycentric space-time frame of reference, C is the Solar-System barycenter. If the spacecraft light-time solution is performed in the local geocentric space-time frame of reference, C refers to the Earth E. The partial derivatives of the computed value z of an angular observable with respect to the space-fixed position vector $\mathbf{r}_3^{\mathrm{C}}(t_3)$ of the receiving station on Earth at the reception time t_3 and the space-fixed position vector $\mathbf{r}_2^{\mathrm{C}}(t_2)$ of the spacecraft at the transmission time t_2 are derived in Subsection 13.9.2.2. The partial derivatives of these space-fixed position vectors with respect to the parameter vector $\mathbf{q}$ are calculated as described in Sections 12.2 and 12.3. These partial derivatives are used in Eq. (12–12) to calculate the partial derivative of the transmission time t_2(ET) at the spacecraft with respect to the parameter vector $\mathbf{q}$.

The partial derivatives of the computed value z of an angular observable with respect to the a, b, and c quadratic coefficients of the time difference $(\mathrm{UTC} - \mathrm{ST})_{t_3}$ at the reception time t_3 at the receiving station on Earth are given by:

$$\frac{\partial z}{\partial a} = \frac{\partial z}{\partial \mathbf{r}_3^{\mathrm{C}}(t_3)} \dot{\mathbf{r}}_3^{\mathrm{C}}(t_3) + \frac{\partial z}{\partial \mathbf{r}_2^{\mathrm{C}}(t_2)} \dot{\mathbf{r}}_2^{\mathrm{C}}(t_2) \left(1 - \frac{\dot{r}_{23}}{c} \right) \tag{13–186}$$

$$\frac{\partial z}{\partial b} = \frac{\partial z}{\partial a} \left(t_3 - t_0 \right) \tag{13–187}$$

$$\frac{\partial z}{\partial c} = \frac{\partial z}{\partial a}\left(t_3 - t_0\right)^2 \tag{13–188}$$

where t_0 is the start time of the time block for the quadratic coefficients which contains the reception time t_3. These epochs can be in station time ST or Coordinated Universal Time UTC. The down-leg range rate $\dot{r}_{23}$ is calculated in the light-time solution using Eqs. (8–56) to (8–59).

The partial derivatives of the computed values of angular observables with respect to the solve-for rotations of the reference coordinate system (to which the angular observables are referred) are given by the coefficients of these rotations in Eqs. (9–55), (9–56) and (9–60) to (9–65).

The computed values of angular observables are referred to the unit vectors **P**, **Q**, and **E** in Figure 9–1 and **N**, **E**, and **Z** in Figures 9–3 to 9–5. These unit vectors are functions of the coordinates of the tracking station on Earth. A one meter change in the station location will change the angular orientation of these unit vectors and hence the computed values of angular observables by about 0.00001 degree, which is negligible in relation to the accuracy of about 0.001 degree for angular observables. Hence, the partial derivatives of the computed values of angular observables with respect to this particular effect of changes in the station coordinates are ignored.

13.9.2.2 Partial Derivatives of Angular Observables With Respect to Position Vectors of Receiving Station and Spacecraft

From Figures 9–1, and 9–3 to 9–5, the partial derivatives of computed values of angular observables with respect to the space-fixed position vector $\mathbf{r}_2^{\mathrm{C}}\left(t_2\right)$ of the spacecraft at the transmission time t_2 are given by:

$$\frac{\partial \lambda_{\mathrm{S/C}}}{\partial \mathbf{r}_2^{\mathrm{C}}\left(t_2\right)} = \frac{{\mathbf{A}_{\mathrm{SF}}}^{\mathrm{T}}}{r_{23}\cos\delta} \tag{13–189}$$

From Figure 9–1 and Eq. (9–41),

$$\frac{\partial HA}{\partial \mathbf{r}_2^{\mathrm{C}}(t_2)} = -\frac{\partial \lambda_{\mathrm{S/C}}}{\partial \mathbf{r}_2^{\mathrm{C}}(t_2)} \tag{13–190}$$

$$\frac{\partial \delta}{\partial \mathbf{r}_2^{\mathrm{C}}(t_2)} = \frac{\mathbf{D}_{\mathrm{SF}}{}^{\mathrm{T}}}{r_{23}} \tag{13–191}$$

$$\frac{\partial \sigma}{\partial \mathbf{r}_2^{\mathrm{C}}(t_2)} = \frac{\tilde{\mathbf{A}}_{\mathrm{SF}}{}^{\mathrm{T}}}{r_{23}\cos\gamma} \tag{13–192}$$

$$\frac{\partial \gamma}{\partial \mathbf{r}_2^{\mathrm{C}}(t_2)} = \frac{\tilde{\mathbf{D}}_{\mathrm{SF}}{}^{\mathrm{T}}}{r_{23}} \tag{13–193}$$

$$\frac{\partial X}{\partial \mathbf{r}_2^{\mathrm{C}}(t_2)} = \frac{\mathbf{A}'_{\mathrm{SF}}{}^{\mathrm{T}}}{r_{23}\cos Y} \tag{13–194}$$

$$\frac{\partial Y}{\partial \mathbf{r}_2^{\mathrm{C}}(t_2)} = \frac{\mathbf{D}'_{\mathrm{SF}}{}^{\mathrm{T}}}{r_{23}} \tag{13–195}$$

$$\frac{\partial X'}{\partial \mathbf{r}_2^{\mathrm{C}}(t_2)} = \frac{\mathbf{A}''_{\mathrm{SF}}{}^{\mathrm{T}}}{r_{23}\cos Y'} \tag{13–196}$$

$$\frac{\partial Y'}{\partial \mathbf{r}_2^{\mathrm{C}}(t_2)} = \frac{\mathbf{D}''_{\mathrm{SF}}{}^{\mathrm{T}}}{r_{23}} \tag{13–197}$$

where the superscript T indicates the transpose of the column unit vector. Changing the signs of these partial derivatives gives the corresponding partial derivatives of computed values of angular observables with respect to the space-fixed position vector $\mathbf{r}_3^{\mathrm{C}}(t_3)$ of the receiving station on Earth at the reception time t_3:

$$\frac{\partial\, angle}{\partial \mathbf{r}_3^{\mathrm{C}}(t_3)} = -\frac{\partial\, angle}{\partial \mathbf{r}_2^{\mathrm{C}}(t_2)} \tag{13–198}$$

In these equations, the unit vectors $\mathbf{A}$, $\mathbf{D}$, $\tilde{\mathbf{A}}$, $\tilde{\mathbf{D}}$, $\mathbf{A}'$, $\mathbf{D}'$, $\mathbf{A}''$, and $\mathbf{D}''$ are calculated as described in Section 9.2.6 and transformed from Earth-fixed rectangular components to space-fixed (subscript SF) rectangular components referred to the celestial reference frame of the planetary ephemeris using Eq. (9–15). The angles in these equations are the computed values of the angular observables and the auxiliary angle $\lambda_{S/C}$. The down-leg range r_{23} is calculated in the spacecraft light-time solution using Eqs. (8–56) to (8–58).